# Riemann $\mathfrak{P}-$ scheme, monodromy and diophantine approximations

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**Abstract**

The aim of this article is devoted to a better understanding of many constructions of effective rational approximations to solutions of some linear hypergeometric functions from the perspective of the monodromy theory.In particular, we give concrete examples of Riemann's ideas rediscovered by G-V Chudnovsky i of the notion of Riemann's module.
(In a modern language, this notion is known as "Fuchsian local system").
We apply this theory to generalize results concerning the simultaneous rational approximations of polylogarithmic functions given by many authors and we effectively construct the system of Padé or Padé type approximants of first -kind at $z = \infty$ of the family $\mathcal{S}$ defined for $1 \leq k \leq q$ by the Lerch's functions

$$\Phi_k(x, z) = \sum_{n=1}^{\infty} \frac{(1/z)^n}{(n+x)^k}.$$

(defined here at $z = \infty$ ) and the Hurwitz zeta-function $\zeta(k, x) = \Phi_k(x, 1)$.

To obtain , almost without calculations such explicit rational approximations, we solve in this particular case a "Riemann-Hilbert problem ":

In addition we give some applications to diophantine approximations.