GKZ, GG AND GK SYSTEMS AND MULTIPLE ZETA VALUES

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August 29, 2012

In papers [1], [2], [4] and [5] Israel Gelfand, his co-workers and his School developed general theory of Hypergeometric functions. It generalizes the theory of Euler-Gauss, General, Appell, Lauricella, Aomoto and many other hypergeometric functions.

Later Gelfand, Graev and Kapranov introduced generalizations of GKZ systems corresponding to the action of nonintegral latices [6], [3] (and also non-compact dual groups) and to noncommutative, reductive group [7].

I'm going to talk about the latest approach to multiple zeta functions, generalizing Rimeann Zeta function ζ, i.e. the functions of the form

$$\zeta(s_1, s_2, ..., s_p) := \sum_{n_1 > n_2 > ... > n_p > 0} n_1^{-s_1} n_2^{-s_2} \dots n_p^{-s_p}, \tag{1}$$

whenever the series converges, from the point of view of Gelfand's School theory of GKZ systems and their generalizations.

References

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[4] Gelfand I., Kapranov M., Zalevinsky A. V., *Equations of hypergeometric type and Newton polytopes.* Soviet Math. Dokl., 37 (1988), p. 678 - 683.

[5] Gelfand I., Kapranov M., Zalevinsky A. V., *Hypergeometric functions and toric varieties.* Funct. Anal. and Appl. 23 no. 2 (1989), 12 - 26.

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