Ergodic Methods in Dynamics

23–27 April 2012, Będlewo, Poland

Titles and abstracts of talks

Jon Aaronson (Tel Aviv University)

IP-rigidity and eigenvalue groups

(joint work with Maryam Hosseini and Mariusz Lemańczyk)

We examine the class of increasing sequences of natural numbers which are IP-rigidity sequences for some weakly mixing mpt. This property is related to certain subgroups of the circle including eigenvalue groups of dyadic towers over the adding machine. There will be examples, including a super-lacunary sequence which is not IP-rigid.

Kari Astala (University of Helsinki)

Thermodynamical formalism, holomorphic motions and related multifractal spectra

We consider the combined stretching and rotational multifractal spectra of holomorphic motions, and determine the universal bounds valid for all motions.

Anna Miriam Benini (CRM, Scuola Normale Superiore Pisa)

Rigidity for non-recurrent parameters in the exponential family

An exponential map $f(z) = e^z + c$ is called nonrecurrent if the asymptotic value c is not in the accumulation set of its own forward orbit. We will present the result that whenever two nonrecurrent exponential maps satisfy some combinatorial equivalence, then they are quasiconformally conjugate. If, moreover, c has a bounded orbit, the conjugation can be made conformal.

Andrzej Biś (University of Lodz)

An analogue of the Variational Principle for group actions

In general, there are many examples of finitely generated groups of homeomorphisms that do not admit any non-trivial invariant measure. Brin and Katok [1] consider a compact metric space (X, d) with a continuous mapping $f : X \to X$ preserving a Borel probability non-atomic measure m and define a local measure entropy $h_m(f, x)$ of f at $x \in X$. We generalize the notion of local measure entropy for the case of a group of homeomorphisms of a metric space and we introduce an upper local measure entropy $h^G_\mu(x)$ and a lower local measure entropy $h_{\mu,G}(x)$ of a group G with respect to a measure μ. We apply the theory of C-structures, elaborated by Pesin in [3], to construct a dimensional type entropy-like invariant and we prove that it coincides with the topological entropy of a group in the sense of [2]. This approach allows us to obtain an analogue of the variational principle for group actions, we obtain lower and upper estimations of the topological entropy of a group by its local measure entropies.

[1] M. Brin and A. Katok, *On local entropy*, in Geometric Dynamics, Lecture Notes in Math. Vol. 1007, Springer, Berlin (1983), 30–38.

[2] E. Ghys, R. Langevin and P. Walczak, *Entropie géométrique des feuilletages*, Acta Math. 160 (1988), 105–142.

[3] Ya. Pesin, *Dimension Theory in Dynamical Systems*, Chicago Lectures in Mathematics, The University of Chicago Press, Chicago, 1997.

Jozef Bobok (Czech Technical University in Prague)

The topological entropy of Banach spaces

In our talk based on [1] we investigate some properties of (universal) Banach spaces of real functions in the context of topological entropy. Among other things, we show that any subspace of $C([0,1])$ which is isometrically isomorphic to ℓ_1 contains a functions with infinite topological entropy. Also, for any $t \in [0, \infty]$, we construct a (one-dimensional) Banach space in which any nonzero function has topological entropy equal to t.

[1] J. B. and H. Bruin, *The topological entropy of Banach spaces*, J. Difference Equ. Appl. 18 (4) (2012), 569–578.

Manfred Denker (Pennsylvania State University)

On the central limit theorem in dynamical systems II

About 25 years ago Feliks organised a meeting at the Banach Center in Warsaw on dynamical systems; a good opportunity to talk at this meeting about the same topic as I did then, and to honor Feliks for his continuing commitment to the subject in research and scientific leadership. I will touch several subjects concerning the interplay of probability theory and dynamics.

Robert Devaney (Boston University)

Mandelpinski necklaces: Structures in the parameter plane for singularly perturbed rational maps

In this lecture we consider rational maps of the form $z^n + C/z^n$ where $n > 2$. When C is small, the Julia sets for these maps are Cantor sets of circles and the corresponding region in the C-plane (the parameter plane) is the McMullen domain. We shall show that the McMullen domain is surrounded by infinitely many simple closed curves called Mandelpinski necklaces. The k^{th} necklace contains exactly $(n-2)n^k + 1$ parameters that are the centers of baby Mandelbrot sets and the same number of parameters that are centers of Sierpinski holes, i.e., disks in the parameter plane where the corresponding Julia sets are Sierpinski curves (sets that are homeomorphic to the Sierpinski carpet fractal). We shall also briefly describe other interesting structures in the parameter plane.

Lorenzo J. Díaz (Pontifícia Universidade Católica do Rio de Janeiro)

Robust vanishing of all central Lyapunov exponents

(joint work with Jairo Bochi and Christian Bonatti)

We describe C^2-open sets of iterated function systems on arbitrary compact manifolds admitting fully supported ergodic measures all whose Lyapunov exponents vanish. We also exploit the consequences for partially hyperbolic maps.

Matúš Dirbák (Matej Bel University)

Extensions of flows by continuous cocycles

We are interested in the existence of a continuous cocycle over a given flow with values in a given topological group, satisfying a given property (topological ergodicity, weak topological mixing, a prescribed set of essential values etc.). We give sufficient conditions which guarantee the existence of such cocycles and apply obtained results to construct minimal fibre-preserving flows on fibre bundles.

Tomasz Downarowicz (Wrocław University of Technology)

Chaos in ergodic systems

We will define a new isomorphism invariant which we call measure-theoretic chaos. It is an analog of distributional chaos appearing in topological dynamics, but defined for measure-theoretic (ergodic) systems without topological structure. Yet, in topological systems this measure-theoretic chaos implies topological distributional chaos. Moreover, our chaos is present in every ergodic system with positive (Kolmogorov–Sinai) entropy. The choice of the subject reflects Feliks Przytycki's latest interest in chaos and similar phenomena (mainly for interval maps). Of course, our notion applies to interval maps as well.

Christophe Dupont (Université Paris-Sud 11)

On Levi-flat hypersurfaces in surfaces of general type

(joint work with Bertrand Deroin)

Levi-flats in complex surfaces are real hypersurfaces foliated by holomorphic curves. I will give examples and present tools for the study of the dynamics of the foliation, namely Garnett's harmonic measures and Lyapunov exponents. Our goal is to use these objects to prove geometrical results. For instance we obtain the following: the unitary tangent bundle of a compact surface of genus ≥ 2 can not be realized as a Levi-flat hypersurface in any (complex) surface of general type. The proof involves dynamics, topology and complex geometry.

Núria Fagella (Universitat de Barcelona)

Connectivity of Julia sets of meromorphic maps with Baker domains

(joint work with Krzysztof Barański, Xavier Jarque and Bogusława Karpińska)

In this talk we show that if a meromorphic transcendental map has a multiply connected Baker domain, then it must also have at least one weakly repelling fixed point (i.e. repelling or with derivative equal to one). This was the last remaining case in the proof of the following result (which was proven by Shishikura for rational maps): If f is a meromorphic transcendental map with a disconnected Julia set, then f has a weakly repelling fixed point. The historical motivation of this theorem was its corollary, namely that the Julia set of Newton's method of every entire map is connected or, equivalently, all its Fatou components are simply connected. To prove this theorem we use a result explained in Xavier Jarque's talk, which shows the existence of absorbing regions in Baker domains, a question which has been open for some time.

Krzysztof Frączek (Nicolaus Copernicus University)

Ergodic properties of translation flows on $\mathbb{Z}$-periodic translation surfaces

(joint work with Corinna Ulcigrai)

A natural motivation to study infinite translation surfaces comes from billiards. As linear flows on compact translation surfaces arise by unfolding billiard flows in rational polygons, flows on infinite translation surfaces can be obtained by unfolding periodic rational billiards, for example the Ehrenfest wind-tree model. Infinite translation surfaces obtained in this way are $\mathbb{Z}^2$(or $\mathbb{Z}$)-covers of compact translation surfaces.

In the classical set-up, a celebrated result, by Kerchoff–Masur–Smillie, states that for every compact connected translation surface (for every rational compact polygonal billiard) the directional flows are ergodic for a.e. direction.

In contrast, I will present some examples of infinite billiards on $\mathbb{Z}^2$(or $\mathbb{Z}$)-periodic polygons for which the set of ergodic directions has measure zero.

Katrin Gelfert (Universidade Federal do Rio de Janeiro)

Thermodynamic formalism in non-hyperbolic dynamics

(joint work with Lorenzo Díaz and Michał Rams)

We aim for an understanding of the thermodynamic formalism in the context of smooth non-hyperbolic dynamics. We follow a classical approach to analyze its basic pieces and study homoclinic classes together with their thermodynamic properties. Focusing on a simple, by representative, example, we construct a local diffeomorphism that is a step skew product modelled over a horseshoe map that is naturally associated to a heterodimensional cycle. This cycle gives rise to a homoclinic class on which the diffeomorphism is topologically transitive and partially hyperbolic. It can be conveniently studied in terms of an iterated function system of interval maps that are genuinely non-contracting. Our examples have topologically a rich fibre structure. Moreover, they exhibit a rich phase transition in the pressure function (coexistence of equilibrium states with positive entropies) that is associated to the Lyapunov exponents in the central direction. This phase transition is a consequence of a gap in the spectrum of central exponents.

Jacek Graczyk (Université Paris-Sud 11)

Metric properties of mean wiggly continua

(joint work with Peter Jones and Nicolae Mihalache)

Eugene Gutkin (Nicolaus Copernicus University)

Billiard caustics, floating in equilibrium, and the isoperimetric inequality

I will discuss a few relationships between these topics.

Yonatan Gutman (Polish Academy of Sciences)

Topological dynamical embedding and Jaworski-type theorems

Given a metric space X of dimension d it is a classical fact that the minimal n that guarantees that X can be embedded in $[0,1]^n$, is $n = 2d+1$. An analogous problem in the category of dynamical systems, is under what conditions one can guarantee that the metric topological system (X,T) is embeddable in $([0,1]^n)^{\mathbb{Z},shift}$. The embedding is induced by n real continuous functions on X, so this question can also be thought as a topological dynamical analogue of the Krieger Generator Theorem. A well known theorem by Jaworski states that if (X,T) is aperiodic and X is finite dimensional then $n=1$ is sufficient. I will discuss several generalizations of this theorem to the infinite-dimensional setting.

Irene Inoquio (Universidade de São Paulo)

Hausdorff measure of the invariant set of a deterministic random walk

(joint work with Daniel Smania)

We study a random walk F generated by a linear expanding map with two branches, and a function that is constant on each interval of monotonicity. Using some probability tools and the thermodynamic formalism, we prove that the t-Hausdorff measure of the invariant set of F is equals to zero, where $t > 0$ is its Hausdorff dimension.

Xavier Jarque (Universitat de Barcelona)

On the existence of absorbing domains and Baker domains

(joint work with Krzysztof Barański, Núria Fagella and Bogusława Karpińska)

Let U be a hyperbolic domain in $\mathbb{C}$ and let $f : U \to U$ be a holomorphic map. An invariant domain $W \subset U$ is called *absorbing in* U *for* f if for every compact set $K \subset U$ there exists $n = n(K) > 0$, such that $f^n(K) \subset W$. The problem of existence of absorbing domains for a given f and U has a long history and it became a useful tool in some dynamical questions.

Based in the Denjoy–Wolff Theorem (on dynamics of holomorphic maps on the unit disc), Cowen proved the existence of a simply connected absorbing domain $V \subset \mathbb{H}$ for holomorphic maps $G : \mathbb{H} \to \mathbb{H}$ (where $\mathbb{H}$ denotes the right half plane) such that $G^n \to \infty$ as $n \to \infty$. The result also gives a (semi)–conjugacy (a conjugacy on V) of the map G with a Möbius transformation T acting on Ω where $\Omega \in \{\mathbb{H}, \mathbb{C}\}$.

Later, König used Cowen's Theorem to prove the existence of simply connected absorbing domains in Baker domains of meromorphic maps with finitely many poles. Moreover, König also showed, by means of an example, that simply connected absorbing domains do not always exist.

The main result we present here is the existence of (possibly multiply connected) absorbing domains in the general case (putting especial attention to the case of Baker domains for transcendental meromorphic functions). In another talk, Núria Fagella will show how to use this tool to prove the connectedness of the Julia set for transcendental meromorphic maps having no weakly repelling fixed points.

Antti Käenmäki (University of Jyväskylä)

Local conical dimensions for measures

We study the decay of $\mu(B(x,r)\cap C)/\mu(B(x,r))$ as $r \downarrow 0$ for different kinds of measures μ on $\mathbb{R}$ and various cones C around x. As an application, we provide sufficient conditions implying that the local dimensions can be calculated via cones almost everywhere.

Henna Lotta Loviisa Koivusalo (University of Oulu)

Hausdorff dimension of affine random covering sets on torus

(joint work with Esa Järvenpää, Maarit Järvenpää, Bing Li and Ville Suomala)

Given a sequence (g_n) of subsets of d-torus $\mathbb{T}^d$ and a sequence of independent, uniformly distributed random variables (ξ_n) on $\mathbb{T}^d$, define the *random covering set* as $E = \limsup_{n\to\infty}(g_n + \xi_n)$. Different properties of random covering sets have been widely studied (for an overview, see [3]).

We calculate the almost sure Hausdorff dimension of a particular type of random covering sets. In the case $d = 1$ this problem was first solved by Fan and Wu in [2]. We consider the case of *affine random covering sets* in $\mathbb{T}^d$, where $g_n = T_n(R)$, (T_n) is a sequence of linear maps and R is any subset of $\mathbb{T}^d$ with nonempty interior. Our dimension formula concerns the singular value function and holds when the sequences of singular values $\alpha_i(T_n)$ decrease to 0 with n for all $i = 1, \ldots, d$.

The upper bound for the dimension follows from a simple covering argument. Following ideas of Fan and Wu in [2] we construct a random Cantor-type subset of E, and use potential theoretic methods in the spirit of Falconer [1] to obtain lower bounds for the dimension.

[1] K. Falconer, *Hausdorff dimension of self-affine fractals*, Math. Proc. Cambridge Phil. Soc. 100 (1988), 339–350.

[2] A. H. Fan and J. Wu, *On the covering by small random intervals*, Ann. I. H. Poincaré Probab. Statist. 40 (2004), 125–131.

[3] J. P. Kahane, *Random coverings and multiplicative processes*, in: Fractal Geometry and Stochastics II (Ch. Bandt, S. Graf and M. Zähle, Eds.), Progress in Probability 46, Birkhäuser, 2000.

Marcin Kulczycki (Jagiellonian University)

On the interplay between ASP and AASP in noncompact case

The average shadowing property (ASP) is an established concept in topological dynamics. Its close relative, the asymptotic average shadowing property (AASP) has been introduced in 2007 and has proven to be a puzzling and hard-to-analyze property. At the moment, in the class of compact spaces, it is not known if either of these properties implies the other. This short talk will discuss the noncompact case.

Dominik Kwietniak (Jagiellonian University)

Properties of systems with the generalized shadowing or specification property

(joint work with Marcin Kulczycki and Piotr Oprocha)

Theories of shadowing and specification, originating with the works of Anosov and Bowen have been developing parallel with the theory of hyperbolic systems. In some crude sense, one may say that these notions are similar. The common goal is to find a true trajectory near an approximate one, but they differ in understanding what constitutes the approximate trajectory. In shadowing one traces a pseudo-orbit, while in specification arbitrarily assembled finite pieces of orbits are supposed to be followed by a true orbit. A template definition for any generalization of shadowing (or specification) might be: for every epsilon there is a delta such that every approximate orbit resembling a true trajectory with an error not greater than delta, can be traced epsilon close by a true one. That template was a base for the subsequent generalizations of both notions.

Blank introduced the notion of average pseudo-orbits and he proved that for a certain kind of perturbed hyperbolic system (X, f), every average pseudo-orbit of f is shadowed on average by some true orbit of f (that is, f has the average shadowing property). Average pseudo-orbits arise naturally in the realizations of independent Gaussian random perturbations with zero mean of hyperbolic systems and in the investigations of the most probable orbits of the dynamical system with general Markov perturbations.

Recently, Climenhaga and Thompson, inspired by the work of Pfister and Sullivan, examined some properties of systems with the almost specification property, which generalizes the notion of specification.

During my talk I shall discuss recurrence properties of systems with the average shadowing property and/or almost specification property. It turns out that almost specification implies average shadowing. Moreover, every dynamical system with the average shadowing property and full invariant measure is topologically weakly mixing. Without the assumption on the support of invariant measure, there is no recurrence property that is implied by the almost specification. We also discuss our result showing that a system $f \colon X \to X$ has the average shadowing property provided f restricted to its measure center has that property.

Mariusz Tomasz Lemańczyk (Nicolaus Copernicus University)

Reversibility and self-similarity of flows

Genadi Miron Levin (Hebrew University of Jerusalem)

Polynomial dynamics and external rays

We give a survey of some works on the subject going back to joint papers with Alex Eremenko (1989) and with Feliks Przytycki (1996).

Bing Li (University of Oulu)

Hitting probabilities of the random covering sets

(joint work with Yimin Xiao and Narn-Rueih Shieh)

By applying the method of limsup type random fractals, as illustrated in Khoshnevisan, Peres and Xiao (2000), we determine the hitting probabilities of the Dvoretzky random covering sets on the circle. Our result also provides an alternative way to obtain the Hausdorff and packing dimensions of such sets, which is different from the methods of Durand (2010).

Lingmin Liao (Université Paris-Est Créteil)

Set of numbers uniformly well-approximated in the sense of Dirichlet

(joint work with Dong Han Kim)

For a fixed irrational x, we consider the numbers y satisfying that for any number $Q > 1$, there exists an integer n less than Q, such that the distance from $nx - y$ to the nearest integer is less than Q^{-s}, with $s > 1$. According to the classic uniform Dirichlet Theorem in Diophantine approximation, we say that these numbers are uniformly well-approximated in the sense of Dirichlet. For any $s > 1$, the Hausdorff dimension of the set of these numbers is obtained and is shown to depend on the Diophantine type of x.

Cristina Lizana Araneda (Universidad de Los Andes / Pontifícia Universidade Católica do Rio de Janeiro)

Robust transitivity for endomorphisms

(joint work with Enrique Pujals)

We address the problem about under what conditions an endomorphism having a dense orbit, verifies that a sufficiently close perturbed map also exhibits a dense orbit. In this direction, we give sufficient conditions, that cover a large class of examples, for endomorphisms on the $n-$dimensional torus to be robustly transitive: the endomorphism must be volume expanding and any large connected arc must contain a point such that its future orbit belong to an expanding region.

Grzegorz Łukaszewicz (University of Warsaw)

Invariant measures for dissipative systems and generalised Banach limits

We show that the generalised Banach limit can be used to construct invariant measures for continuous dynamical systems on metric spaces that have compact attracting sets, taking limits evaluated along individual trajectories. We also show that rather than taking limits evaluated along individual trajectories, we can take an ensemble of initial conditions: the generalised Banach limit can be used to construct an invariant measure based on an arbitrary initial probability measure, and any invariant measure can be obtained in this way.

The motivation for this research comes from considerations of an infinite dimensional dynamical system associated with the Navier–Stokes equations and of turbulence problems.

[1] G. Łukaszewicz, J. Real and J. C. Robinson, *Invariant measures for dissipative systems and generalised Banach limits*, J. Dyn. Diff. Equat. 23 (2011), 225–250.

Anthony Manning (University of Warwick)

A map of the tetrahedron that describes the sequence of pedal triangles

Any triangle ABC has a pedal triangle (whose vertices are the feet of the perpendiculars from each vertex to the opposite side). We study how the shape changes as we repeatedly take the pedal triangle. Does the sequence of triangles converge and, if so, how does the limit depend on the original triangle?

Wacław Marzantowicz (Adam Mickiewicz University)

Some topological tools in dynamical systems

In seventies of twentieth century in dynamical systems appeared problems which are of topological nature (local and global) and need also topological tools to study them. They are connected with estimations of the rate of growth of number of periodic points or the topological entropy. We will give a short survey of some investigations in this direction from their origins to recent results.

Eugen Mihailescu (Romanian Academy)

Ergodic and metric properties for smooth systems with overlaps

We will study several classes of non-invertible dynamical systems, both from the metric point of view (Hausdorff dimension, sectional dimensions, etc.) and from the ergodic point of view (1-sided Bernoullicity of certain measures, mixing). We look at a class of hyperbolic skew products which have strong non-invertibility character reflected in the fiberwise stable dimension. For such maps it is possible to estimate the angle between various intersecting unstable manifolds and the stable/unstable dimensions. Another class will be given by toral endomorphisms and toral extensions satisfying certain cocycle conditions. For non-invertible systems one can investigate also equilibrium measures given by Hölder potentials, and their conditional measures.

Michał Misiurewicz (Indiana University – Purdue University Indianapolis)

Random interval homeomorphisms

(joint work with Lluís Alsedà)

We investigate two interval homeomorphisms, one moving points to the right and one to the left, applied randomly. We consider this system as a skew product with the Bernoulli shift in the base. Both homeomorphisms are piecewise linear and both endpoints are repelling in average. We prove that for almost all fibers the map is basically a contraction. If the shift in the base is two-sided, we prove the existence of a global (a.e.) pullback attractor, which is the graph of a function from the base to the fiber space. It is also a usual (forward) attractor. However, when we replace the map in the base by the one-sided shift, the attractor vanishes and instead of it we get invariance of the Lebesgue measure. This is a paradox, since forgetting about the past should not change the future behavior of the system. We call this phenomenon the mystery of the vanishing attractor.

Viêt Anh Nguyên (Université Paris-Sud 11)

Exponential estimates for plurisubharmonic functions and stochastic dynamics

(joint work with Tien Cuong Dinh and Nessim Sibony)

We discuss several stochastic properties for the equilibrium measures associated to holomorphic maps on projective spaces $\mathbb{CP}^k$. More precisely, we prove the exponential decay of correlations, the central limit theorem for general d.s.h. observables, and the large deviations theorem for bounded d.s.h. observables and Hölder continuous observables. The key technique is some exponential estimates for plurisubharmonic functions with respect to Monge–Ampère measures with Hölder continuous potential.

Daniel Nicks (University of Nottingham)

Iteration of quasiregular tangent functions in three dimensions

(joint work with Alastair Fletcher)

We define a new quasiregular mapping $T : \mathbb{R}^3 \to \mathbb{R}^3 \cup \{\infty\}$ that generalizes the tangent function on the complex plane and shares a number of its geometric properties. We investigate the dynamics of the family $\{\lambda T : \lambda > 0\}$, establishing results analogous to those of Devaney and Keen for the meromorphic family $\{z \mapsto \lambda \tan z : \lambda > 0\}$.

Magdalena Nowak (Jagiellonian University / Jan Kochanowski University in Kielce)

Topological classification of countable IFS-attractors

We deal with the part of Fractal Theory related to finite families of contractions called iterated function systems (IFS). An attractor is a compact set, invariant for such a family F. In other words, an IFS-attractor is the unique fixed-point of the natural contraction (induced by F) acting on the hyperspace of non-empty compact sets endowed with the Hausdorff metric. It is easy that any geometric convergent sequence is an IFS-attractor. On the other hand, it can be topologically embedded in the real line so that its image is not an IFS-attractor. We show that each scattered space of successor height is homeomorphic to an attractor of IFS. On the other hand, we show that a countable compact scattered space of limit height cannot be an IFS-attractor in any metric.

Tomasz Nowicki (IBM Research)

Error Diffusion on simplices: invariant regions, tessellations and acuteness

(joint work with Roy Adler, Grzegorz Świrszcz, Charles Tresser and Shmuel Winograd)

The error diffusion algorithm can be considered as a time dependent dynamical system that transforms a sequence of *inputs* into a sequence of *inputs*. That dynamical system is a time dependent translation acting on a partition of the phase space Φ, an affine space, into the Voronoï regions of the set C of vertices of some polytope Pol where the inputs all belong.

Given a sequence $g(i)$ of inputs that are point in Φ, $g(i)$ gets added to the *error vector* $e(i)$, the total vector accumulated so far, that belongs to the vector space ϕ associated to Φ. The sum $g(i) + e(i)$ is then again in Φ, thus in a well defined element of the partition of Φ that determines in turns one vertex $v(i)$. The point $v(i)$ of Φ is the i^{th} output, and the new error vector to be used next is $e(i+1) = g(i) + e(i) - v(i)$. The maps $e(i) \mapsto e(i+1)$ and $g(i)+e(i) \mapsto g(i+1)+e(i+1)$ are two form of error diffusion, respectively in ϕ and Φ. Long term behavior of the algorithm can be deduced from the asymptotic properties of invariant sets, especially from the absorbing ones that serve as traps to all orbits. The existence of invariant sets for arbitrary sequence of inputs has been established in full generality, but in such a context, the invariant sets that are shown to exist are arbitrarily large and only few examples of minimal invariant sets can be described. Since the case of constant input (that corresponds to a time independent translation) has its own interest, we study here the invariant set for constant input for special polytopes that contain the n-dimensional regular simplexes.

In that restricted context of interest in number theory, we study the properties of the minimal absorbing invariant set and prove that typically those sets are bounded fundamental sets for a discrete lattice generated by the simplex and that the intersections of those sets with the elements of the partition are fundamental sets for specific derived lattices.

Piotr Oprocha (Polish Academy of Sciences)

Inverse limits of maps on topological graphs and hereditarily indecomposable continua

(joint work with Piotr Kościelniak and Murat Tuncali)

In 1991 Minc and Trensue presented a method of construction of maps on pseudoarc which are topologically transitive (and hence have positive topological entropy). In 1993 Chen and Li proved that homeomorphism of the pseudoarc constructed by Henderson (in 1964) has shadowing property (and zero topological entropy). They asked if it is possible to construct a map on the pseudoarc with positive topological entropy and shadowing.

The above mentioned result of Minc and Trensue have at last two possible extensions. First, it is possible to modify their technique to answer question of Chen and Li in the affirmative (Kościelniak & Oprocha, 2010) and on the other hand, it is also possible to adopt their ideas to the case of inverse limits of graphs (Kawamura, Tuncali & Tymchatyn, 2005), obtaining transitive homeomorphisms on other types of hereditarily indecomposable continua (e.g. pseudo-circle).

In this talk we will comment on possible combination of the above mentioned extensions of Minc and Trensue technique and further dynamical properties of graph maps obtained as the result of this construction (like topological mixing).

John Osborne (Open University)

Spiders' webs and locally connected Julia sets of transcendental entire functions

In this talk, I will explore some links between the local connectedness of the Julia set of a transcendental entire function, and a particular geometric form of the Julia set known as a spider's web (first defined by Rippon and Stallard).

Maria José Pacifico (Universidade Federal do Rio de Janeiro)

A toy model for flows with equilibria attached to regular orbits

(joint work with Roberto Markarian and José Vieitez)

We shall analyze the C^∞ co-cycle $f : [0,1] \times [0,1] \setminus \{1/2\} \to [0,1] \times [0,1]$ defined as

$$f(x,y) = \left(2x - \lfloor 2x \rfloor, y + \frac{c}{|x-1/2|} - \left\lfloor y + \frac{c}{|x-1/2|} \right\rfloor \right),\ c \in \mathbb{R}^+.$$

We prove that f is topologically mixing and if $c > 1/4$ then f is mixing with respect to Lebesgue measure. Furthermore we prove that the speed of mixing is exponential.

This co-cycle may be seen as a simplified case of the slipping effect in flows with singularities attached to regular orbits. This is the case for a Lorenz-like flow on 3-manifolds.

Liviana Palmisano (Université Paris-Sud 11)

A phase transition for circle maps and Cherry flows

We study C^2 weakly order preserving circle maps with a flat interval. The main result of the paper is about a sharp transition from degenerate geometry to bounded geometry depending on the degree of the singularities at the boundary of the flat interval. We prove that the non-wandering set has zero Hausdorff dimension in the case of degenerate geometry and it has Hausdorff dimension strictly greater than zero in the case of bounded geometry. Our results about circle maps allow to establish a sharp phase transition in the dynamics of Cherry Flows.

Christopher Shaun Penrose (Queen Mary, University of London)

Physical measures for non-autonomous Blaschke products

(joint work with Christian Beck)

Suppose we are given two C^2 orientation-preserving circle maps A and B which both expand Lebesgue measure λ on the unit circle. Given an arbitrary composition $C = C_l \circ \ldots \circ C_1$ (or word) with $C_i \in \{A, B\}$ we obtain the unique invariant probability measure λ_C equivalent to λ.

Now fixing $C_1, \ldots, C_l$ and continuing C_i for $i > l$ periodically by $C_i = C_{i-l}$, we are interested in the the non-autonomous time average $\frac{1}{N}\sum_{i=0}^{N-1} \delta_{C_i \circ \ldots \circ C_1(u)}$ given $u \in \mathbf{S^1}$. For λ-almost all u this converges, as $N \to \infty$, in the weak-* topology to the *Birkhoff measure* $\frac{1}{l}\sum_{k=0}^{l-1} \lambda_{C_k \circ \ldots \circ C_1 \circ C_l \circ \ldots \circ C_{k+1}}$.

We study the special case where $C_i = A$ for alternate blocks of i of length m and $C_i = B$ for intervening blocks of i of length n. We thus obtain the Birkhoff measure

$$\frac{1}{m+n}\left\{\sum_{i=1}^{m} \lambda_{A^i \circ B^n \circ A^{m-i}} + \sum_{j=1}^{n} \lambda_{B^j \circ A^m \circ B^{n-j}}\right\} .$$

We are interested in the statement that as m and n tend to infinity with fixed ratio $p : q$ (satisfying $p + q = 1$) this Birkhoff measure tends to the super-statistical limit $p\lambda_A + q\lambda_B$. We examine this convergence in the special case of A and B degree two Blaschke products.

Tomas Persson (Lund University)

On the Diophantine properties of λ-expansions

(joint work with Henry Reeve)

For $\lambda \in (1/2, 1)$ and $\alpha > 1$, we consider sets of numbers x such that for infinitely many n, x is $2^{-\alpha n}$-close to some $\sum_{i=1}^{n} \omega_i \lambda^i$, where $\omega_i \in \{0, 1\}$. These sets are in Falconer's intersection classes for Hausdorff dimension s for some s such that $-\frac{1}{\alpha}\frac{\log \lambda}{\log 2} \leq s \leq \frac{1}{\alpha}$. We show that for almost all $\lambda \in (1/2, 2/3)$, the upper bound of s is optimal, but for a countable infinity of values of λ the lower bound is the best possible result.

Carsten Lunde Petersen (Roskilde University)

Resurgence of receding limbs

Milnor defined natural coordinates on the Moduli space $\mathcal{M}$ of quadratic rational maps. These coordinates identifies $\mathcal{M}$ with $\mathbb{C}^2$. Moreover, for the standard identification of $\mathbb{CP}^2$ with $\mathbb{C}^2 \cup \mathbb{CP}^1$, Milnors coordinates extends to the space of Möbius transformations and sends them to the compactifying Riemann sphere at infinity. This shows that we may compactify $\mathcal{M}$ by adding the moduli space of Möbius transformations. It was first proved by Adam Epstein, that certain sequences of higher iterates of quadratic rational maps converge to quadratic rational maps with a parabolic fixed point of multiplier 1. In a joint work we prove that in fact any quadratic rational map with a fixed point of multiplier 1 and connected Julia set can be obtained this way.

Mark Pollicott (University of Warwick)

Dynamical zeta function

Peter Raith (Universität Wien)

Hausdorff dimension of invariant subsets of small dimensional dynamical systems

For very simple dynamical systems with $|T'| = \alpha > 1$ it is easy to check that an invariant subset X satisfies $\dim_H X = \frac{h_{\text{top}}(T)}{\log \alpha}$, where $\dim_H$ denotes the Hausdorff dimension and h_{top} denotes the topological entropy. This result has been generalized to a large class of dynamical systems, where it can be shown that the Hausdorff dimension equals the zero of the pressure function. It is known as the Bowen–McCluskey–Manning formula.

Let $T : [0,1] \to [0,1]$ be a piecewise monotonic map, this means there exists a finite partition $\mathcal{Z}$ of $[0,1]$ into finitely many pairwise disjoint open intervals satisfying $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0,1]$ such that $T\big|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. Moreover assume that T is differentiable on $\bigcup_{Z\in\mathcal{Z}} Z$ and $\inf\left\{|T'x| : x \in \bigcup_{Z\in\mathcal{Z}} Z\right\} > 1$ (the abbreviation $\inf|T'|$ will be used for this infimum). Note that T need not be continuous at the endpoints of the intervals of monotonicity. Consider an open $U \subseteq [0,1]$, and define $R := \bigcap_{j=0}^{\infty} \overline{[0,1] \setminus T^{-j}U}$, this means the set of all x whose orbits always omit U. One can show that $\dim_H R$ equals the unique zero of $t \mapsto p(R, T, -t\log|T'|)$, where p denotes the topological pressure.

Using this result it can be proved that the Hausdorff dimension is lower semi-continuous and upper bounds for the jumps up can be given. Moreover, for an ergodic invariant Borel probability measure μ one can prove that $\dim_H \mu = \frac{h_\mu(T)}{\log \mu(|T'|)}$, where $\dim_H \mu$ is the infimum of the Hausdorff dimensions of subsets of full measure, h_μ is the measure-theoretic entropy and $\mu(f) := \int f\, d\mu$. It is also possible to obtain results on multifractal dimensions.

Nonetheless for one-dimensional maps the invariant sets under consideration are repellors, and therefore they are not very interesting. For a class of two-dimensional skew-product maps one obtains an attractor whose Hausdorff dimension satisfies a similar formula.

Michał Rams (Polish Academy of Sciences)

Local dimension spectrum for parabolic maps

(joint work with Thomas Jordan)

We consider a topologically expanding C^1 map of an interval with a parabolic periodic point and investigate the local dimension spectrum for such maps. As Gibbs measures are not natural objects for parabolic systems, we are working with weak Gibbs measures which could be defined for any continuous potential. Our results include description of the spectrum via hyperbolic measures and as Legendre transform of the temperature function.

Henry William Joseph Reeve (University of Bristol)

Multifractal analysis in non-conformal dynamics

Phil Rippon (Open University)

On a conjecture of Baker and a conjecture of Eremenko II

These talks will describe recent work on two conjectures concerning the iteration of a transcendental entire function f: Baker's conjecture that all the components of the Fatou set $F(f)$ are bounded if f has order less than $1/2$ and Eremenko's conjecture that all the components of the escaping set $I(f)$ are unbounded. The proofs of all earlier results on Baker's conjecture in fact imply the stronger result that the fast escaping set $A(f)$ is a 'spider's web'. This implies that $I(f)$ is connected and hence Eremenko's conjecture also holds, giving an unexpected connection between the two conjectures.

In the first talk we give a condition on the growth of the maximum modulus which implies that $A(f)$ is a spider's web. This condition is obtained by proving a local version of the $\cos \pi\rho$ theorem. We also give examples of functions (including some of order zero) for which $A(f)$ is not a spider's web. These examples show that our growth condition is sharp and that new techniques are needed in order to prove Baker's conjecture.

In the second talk we show that both conjectures hold whenever f has order less than $1/2$ and all the zeros of f are on the negative axis. We do this by introducing new techniques which allow us to show that the images of certain curves must wind round the origin. We deduce that, for such functions, $I(f)$ is a spider's web. Our results apply to the examples given in the first talk – these are the first examples of functions for which $I(f)$ is a spider's web but $A(f)$ is not.

Hans Henrik Rugh (Université Paris-Sud 11)

Complex cones in ergodic theory

We describe a recent theory for the projective contraction of complex cones, generalizing Birkhoff's 1957 use of the Hilbert metric to describe real cone contractions. We will discuss applications to dynamical systems and ergodic theory, notably for obtaining central limit theorems for certain types of skew products.

Tuomas Sahlsten (University of Helsinki)

Local entropy averages and the fine structure of measures

(joint work with Pablo Shmerkin and Ville Suomala)

We present a general approach to the study of the local distribution of measures on Euclidean spaces, based on local entropy averages. As concrete applications, we unify, generalize, and simplify a number of recent results on local homogeneity, porosity and conical densities of measures.

Tony Samuel (Universität Bremen)

Spectral metric spaces for Gibbs measures

The common idea of Connes' Non-commutative Geometry is to represent a geometric object by an operator algebra and in doing so one is able to build an analogue of a differential structure for these operator algebras. Connes showed that the starting point in forming such a theory is to form a spectral triple.

In this talk, by way of examples, we will show how one can construct a spectral triple which will represent a sub-shift of finite type Σ equipped with a Gibbs measure μ. Specifically, we will construct a spectral triple from which one can recover both geometry of Σ as well as the measure μ. Since sub-shifts of finite type can be used to represent a wide variety of fractal sets, the theory we will present will allow one to begin to describe an analogue of a differential structure for a wide class of fractal sets.

Omri Sarig (Weizmann Institute of Science)

Bernoulli property for equilibrium measures of surface diffeomorphisms

Suppose f is a $C^{1+\epsilon}$ diffeomorphism on a compact smooth surface M. We show that if an equilibrium measure for a Holder continuous potential on M has positive entropy, then it is

Bernoulli up to a period.

Jörg Schmeling (Lund Institute of Technology)

Multifractal analysis of multiple mixing level sets – first results, problems and questions

Multifractal analysis of multiple mixing is a by far non-trivial generalization of standard multifractal analysis. We will investigate questions like the following. Given a dynamical system $T\colon X \to X$ and a function $F\colon X^d \to \mathbb{R}$ we are interested in evaluating the size of the level sets

$$E(\alpha) = \left\{ x \in X \ : \ \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} F(T^k x, T^{2k} x, \cdots, T^{dk} x) = \alpha \right\}.$$

Surprisingly the packing dimension, the Hausdorff dimension and the maximal dimension of an invariant measure are different in general. Moreover several types of a new kind of phase transition occur.

Daniel Schnellmann (École normale supérieure)

On the Whitney-Holder regularity of the SRB measure in the quadratic family

(joint work with Viviane Baladi and Michael Benedicks)

We consider the set of Collet-Eckmann parameters for the quadratic family and show that the SRB measure depend Holder continuously on the parameter (in the sense of Whitney). Furthermore, we give examples where we have a lower bound which is Hölder-1/2. This result stands in contrast to the case where one has a family of smooth unimodal maps which are in the same topological class: under generic conditions on the family, the SRB measure depends in this case differentiably on the parameter and its derivative is given by an explicit "linear response" formula (Baladi and Smania). Our result indicates that in the transversal case, e.g. in the quadratic family, linear response might not hold.

Karoly Simon (Budapest University of Technology and Economics)

Projections of fractal percolation

(joint work with Michał Rams)

In this talk we study various kinds of projections of a family of random fractals. To study turbulence, B. Mandelbrot introduced a random fractal which is now called now "Mandelbrot percolation" or "fractal percolation". The construction is as follows: we are given an integer $M \geq 2$ and a probability $0 < p < 1$. We partition the unit square $Q = [0,1]^2$ into M^2 congruent sub-squares and we keep each of them with probability p and throw away with probability $1 - p$. In each of the squares retained we repeat the same process at infinitum. Whatever remains after infinitely many steps is the random fractal denoted Λ. This may be an empty set but for $p > 1/M^2$ we have $\Lambda \neq \emptyset$ with positive probability. In particular, for $1/M < p < 1$ the Hausdorff dimension $\dim_{\mathrm{H}}(\Lambda) > 1$ almost surely, conditioned on $\Lambda \neq \emptyset$. In this case, we prove that the orthogonal projection of Λ in every direction contains an interval. We also study projections of fractal percolation in the case when $\dim_{\mathrm{H}} \Lambda < 1$.

Daniel Smania (Universidade de São Paulo)

Renormalization operator for multimodal maps

Renormalization theory in one-dimensional dynamics has been a hot topic along the years, specially after the seminal work of Douady–Hubbard and Sullivan. Perhaps one of the most striking. developments is that a fine understanding of the renormalization operator can lead us a better knowledge of the behavior of most of one-dimensional dynamical systems. For instance, the work of Avila, Lyubich and de Melo on families of real analytic unimodal maps relays deeply on renormalization theory.

A similar approach for multimodal maps (many critical points) pose new difficulties. Mainly the parameter space is not one-dimensional. The parapuzzles, developed by Branner-Hubbard and applied successfully by Yoccoz and many others for unicritical maps, provided a very precise description of the parameter space of the quadratic family. The miraculous properties of codimension one holomorphic laminations were also a crucial tool to understand the space of quadratic-like maps. Both tools are no longer available in the multimodal case. In this work in progress our main result is as follows:

Main Theorem. *Let f_λ be a finite-dimensional family of real analytic multimodal maps and let Λ_b be the subset of parameters λ such that f_λ is infinitely renormalizable with bounded combinatorics (not all the critical points need to be involved in the renormalization). Then for a generic finite-dimensional family the set Λ_b has zero Lebesgue measure.*

One of the main steps of the proof is to show that the action of the renormalization operator on infinitely renormalizable multimodal maps with bounded combinatorics is hyperbolic. The contraction on the hybrid classes of infinitely renormalizable maps can be obtained using available methods (Sullivan, McMullen, Lyubich, S., Lyubich and Avila). To show the expansion in the transversal direction, we developed a new approach, based on the study of the derivative cocycle of the renormalization operator instead of the operator itself...

Gwyneth Stallard (Open University)

On a conjecture of Baker and a conjecture of Eremenko I

These talks will describe recent work on two conjectures concerning the iteration of a transcendental entire function f: Baker's conjecture that all the components of the Fatou set $F(f)$ are bounded if f has order less than 1/2 and Eremenko's conjecture that all the components of the escaping set $I(f)$ are unbounded. The proofs of all earlier results on Baker's conjecture in fact imply the stronger result that the fast escaping set $A(f)$ is a 'spider's web'. This implies that $I(f)$ is connected and hence Eremenko's conjecture also holds, giving an unexpected connection between the two conjectures.

In the first talk we give a condition on the growth of the maximum modulus which implies that $A(f)$ is a spider's web. This condition is obtained by proving a local version of the $\cos \pi\rho$ theorem. We also give examples of functions (including some of order zero) for which $A(f)$ is not a spider's web. These examples show that our growth condition is sharp and that new techniques are needed in order to prove Baker's conjecture.

In the second talk we show that both conjectures hold whenever f has order less than 1/2 and all the zeros of f are on the negative axis. We do this by introducing new techniques which allow us to show that the images of certain curves must wind round the origin. We deduce that, for such functions, $I(f)$ is a spider's web. Our results apply to the examples given in the first talk – these are the first examples of functions for which $I(f)$ is a spider's web but $A(f)$ is not.

Bernd Otto Stratmann (Universität Bremen)

Extreme Value Laws for maximal cuspidal windings

In this talk we present recently obtained Extreme Value Laws for maximal cuspidal windings of the geodesic flow on certain cusped Riemannian surfaces. These results extend previous work by Khintchine, Galambos and Dolgopyat. Moreover, they allow to strengthen previous results by Sullivan, Velani and the speaker for Kleinian groups, as well as earlier work by Philipp on continued fractions, where the latter was inspired by an almost correct conjecture of Erdös.

Jean-Marie Strelcyn (Université de Rouen)

On Chouikha's isochronicity criterion

Recently A. R. Chouikha gave a new characterization of isochronicity of center at the origin for the system of planar differential equations $\dot{x} = y$, $\dot{y} = -g(x)$, where g is a real smooth function defined in some neighborhood of $0 \in \mathbb{R}$. We present some new development of the subject.

Grzegorz Michał Świrszcz (IBM Research)

Disjunctive cuts, lattice-free sets and mixed-integer programming

In the talk we describe the topic of mixed-integer programming. The talk is addressed to a wide audience, no specific knowledge of MIP algorithms is assumed. The results are an outcome of combining forces of domain expert computer scientists and mathematicians not working on MIP problems before. We discuss how this subject is related to geometry of convex sets by means of disjunctive cuts introduced by Li and Richard in 2008. We present new results describing the complexity of such algorithms which lead to interesting geometric questions about lattice-free convex bodies. By analyzing n-dimensional lattice-free sets, we prove that every facet-defining inequality of the convex hull of a mixed-integer polyhedral set with n integer variables is a t-branch split cut for some positive integer t. Moreover, this number t does not depend on the data defining the polyhedral set and is bounded by a function of the dimension n only. We use this result to give a finitely convergent cutting-plane algorithm to solve mixed-integer programs. We also show that the minimum value t, for which all facets of polyhedral mixed-integer sets with n integer variables can be expressed as t-branch split cuts, grows exponentially with n. In particular, when $n = 3$, we observe that not all facet-defining inequalities are 6-branch split cuts. We analyze the cases when $n = 2$ and $n = 3$ in detail, and show that an explicit classification of maximal lattice-free sets is not necessary to express facet-defining inequalities as branching disjunctions with a small number of atoms.

Jean-Paul Thouvenot (Université Pierre et Marie Curie)

On positive entropy transformations

Maciej Wojtkowski (University of Warmia and Mazury in Olsztyn)

Riccati equations in $\mathbb{R}^n$

Motivated by the results of Żołądek on quaternionic Riccati equation we will extend it to any $\mathbb{R}^n$.

Yiwei Zhang (University of Exeter)

On the mixing properties of piecewise expanding maps under composition with permu-

tations

(joint work with Nigel Byott and Mark Holland)

For a mixing and uniformly expanding interval map $f : I \to I$ we pose the following questions. For which permutation transformations $\sigma : I \to I$ is the composition $\sigma \circ f$ again mixing? When $\sigma \circ f$ is mixing, how does the mixing rate of $\sigma \circ f$ typically compare with that of f?

As a case study, we focus on the family of maps $f(x) = mx \bmod 1$ for $2 \le m \in \mathbb{N}$. We split $[0, 1)$ into N equal subintervals, and take σ to be a permutation of these. We analyse those $\sigma \in S_N$ for which $\sigma \circ f$ is mixing, and show that for large N, typical permutations will preserve the mixing property. However, when $\sigma \circ f$ is mixing, we will show the existence of permutations in S_N for which the (exponential) mixing rate of $\sigma \circ f$ can be made arbitrarily close to one (as $N \to \infty$).

Michel Zinsmeister (Université d'Orléans)

Hausdorff dimension of the Julia sets of some real polynomials

(joint work with Genadi Levin)

The supremum of the dimensions of the Julia sets of $z^2 + c$, c real, is not known (the question being of course if it is 2 or not). We prove that if d is an even integer then the sup of the dimensions of the Julia sets of $z^d + c$, c real, is greater than $2d/(d + 1)$.

Henryk Żołądek (University of Warsaw)

Linear meromorphic differential equations and multiple zeta values

The multiple zeta values $\zeta(a_1, \ldots, a_r)$ are natural generalizations of the values $\zeta(a)$ of the Riemann zeta functions at integers $a \ge 2$. They have many applications, e.g. in the knot theory and in the quantum physics. It turns out that some generating functions for the multiple zeta values, like $f_a(x) = 1 - \zeta(a)x^a + \zeta(a, a)x^{2a} - \ldots$, are related with hypergeometric equations. More precisely, $f_a(x)$ is the value at $t = 1$ of the hypergeometric series $g(t; x) =_{a-1} F_a(t) = 1 - x^a t + \ldots$, a solution to the equation $(1 - t)\,\frac{d}{dt}\left(t\frac{d}{dt}\right)^{a-1} g + x^a g = 0$. Our (i.e. mine with M. Zakrzewski) idea is to represent $f_a(x)$ as some connection coefficient between certain standard bases of solutions near $t = 0$ and near $t = 1$. Moreover, we assume that $|x|$ is large. For large complex x the above basic solutions are represented in terms of so-called WKB solutions (introduced by physicists in the case of the Schrödinger equation). The series which define the WKB solutions are divergent and are subject to so-called Stokes phenomenon. Anyway it is possible to treat them rigorously. I will review our results about application of the WKB method to the generating functions $f_2(x)$ and $f_3(x)$.