ASP vs AASP

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A joint work with:

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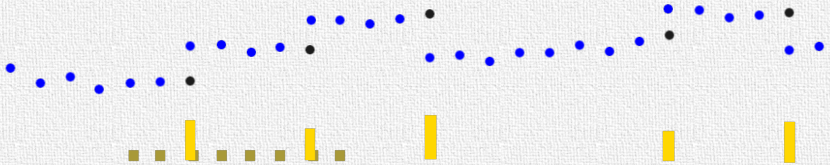
- Piotr Oprocha (AGH University of Science and Technology)

Let (X, d) be a metric space.
Let $f\colon X \mapsto X$ be a continuous function.

Introduced by M. L. Blank in 1988.
Let $\delta > 0$ and let $\{x_n\}_{n=0}^{\infty}$ be a sequence of points from X.

A δ-average pseudo-orbit

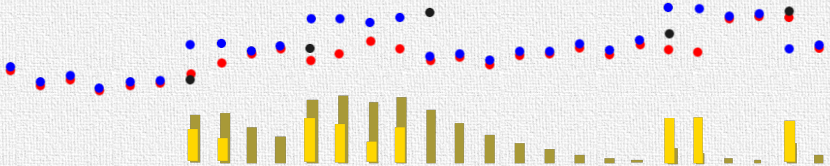
The sequence $\{x_n\}_{n=0}^{\infty}$ is a *δ-average pseudo-orbit of f* if there is an integer $N > 0$ such that for all $n \geqslant N$ and $k \geqslant 0$ we have $\frac{1}{n}\sum_{i=0}^{n-1} d(f(x_{i+k}), x_{i+k+1}) < \delta$.



Let $\varepsilon, \delta > 0$ and $y \in X$. Let $\{x_n\}_{n=0}^{\infty}$ be a δ-average pseudo-orbit of f.

The ε-shadowing in average

The pseudo-orbit $\{x_n\}_{n=0}^{\infty}$ is *ε-shadowed in average by y* if $\limsup_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} d(f^i(y), x_i) < \varepsilon$.



ASP

The map f has the average shadowing property (ASP) if for every $\varepsilon > 0$ there exists $\delta > 0$ such that every δ-average pseudo-orbit of f is ε-shadowed in average by some point in X.

Introduced by R. Gu in 2007.

Articles on AASP

2011 Dyn. Syst. *Flows with the (asymptotic) average...*
2011 J. Adv. Res. Dyn. Control Syst. *The asymptotic average...*
2011 Chaos Solitons Fractals *On strong ergodicity...*
2011 Fund. Math. *Properties of dynamical systems...*
2010 J. Difference Equ. Appl. *Exploring the asymptotic average...*
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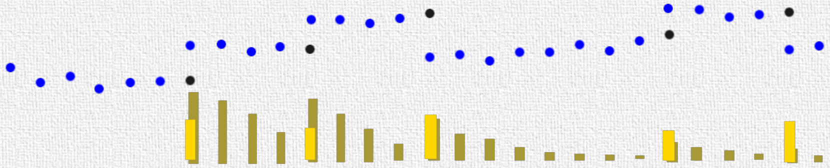
Let $\{x_n\}_{n=0}^{\infty}$ be a sequence of points from X.

An asymptotic-average pseudo-orbit

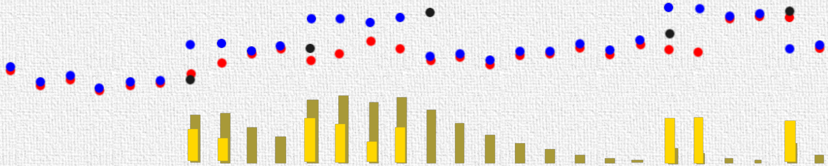
The sequence $\{x_n\}_{n=0}^{\infty}$ is an *asymptotic-average pseudo-orbit of f* if $\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f(x_i), x_{i+1}) = 0$.



Let $y \in X$ and let $\{x_n\}_{n=0}^{\infty}$ be an asymptotic-average pseudo-orbit of f.

The asymptotic shadowing in average

The pseudo-orbit $\{x_n\}_{n=0}^{\infty}$ is *asymptotically shadowed in average by* y if $\limsup_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} d(f^i(y), x_i) = 0$.



AASP

The map f has the asymptotic average shadowing property (AASP) if every asymptotic-average pseudo-orbit of f is asymptotically shadowed in average by some point in X.

So far, all maps analyzed in publications either have both the ASP and the AASP or have neither...

ASP = AASP ?

As it turns out, there exist counterexamples in noncompact spaces

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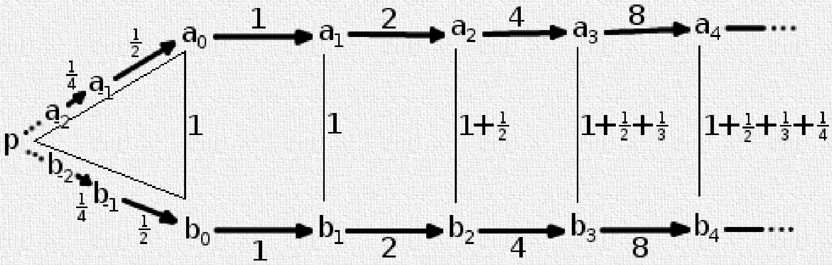
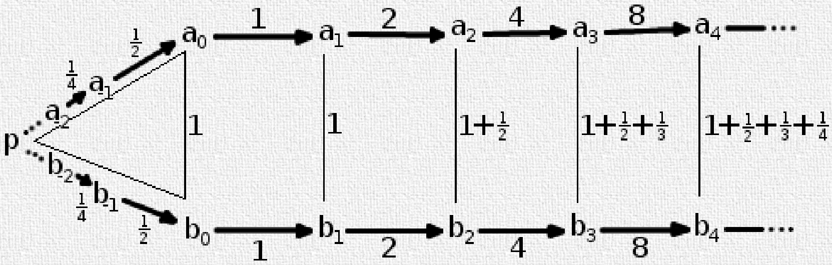


The space X_1 and the map f_1

Theorem 1

The map f_1 has the ASP, but does not have the AASP.

(In preparation)



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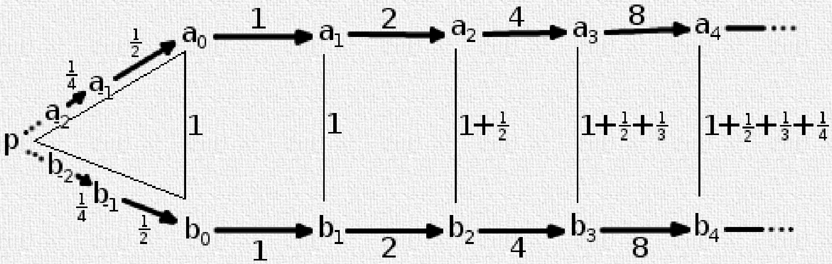

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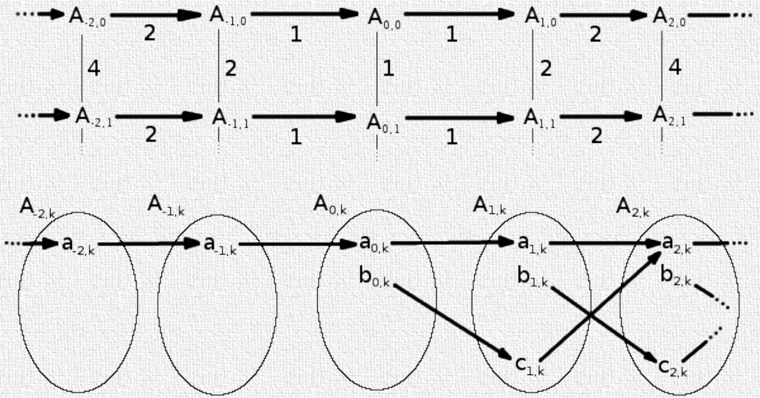

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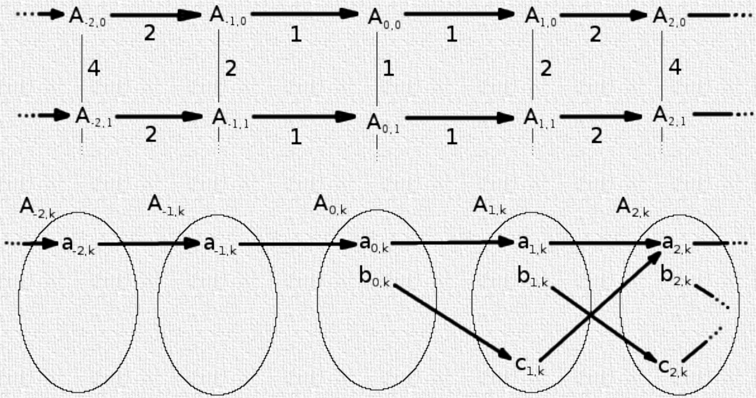

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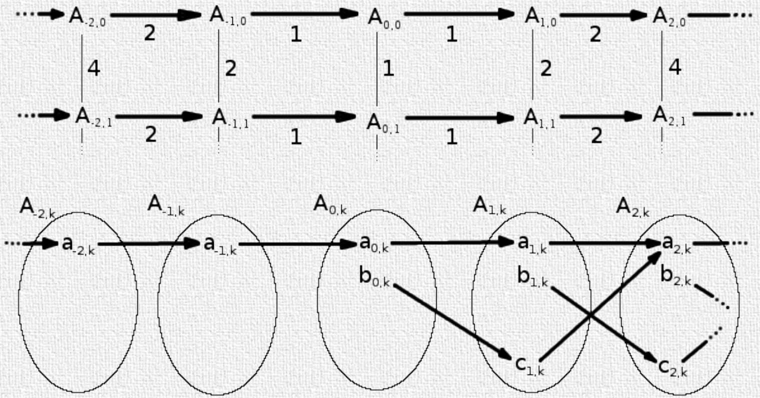

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Theorem 3

If X is compact, then AASP implies ASP.

(In preparation, too)

Question A

Does ASP imply AASP when X is compact?

I think so...

Question B

Are there any implications in the class of bounded metric spaces?

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