Local entropy averages and the fine structure of measures

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University of Helsinki, Finland

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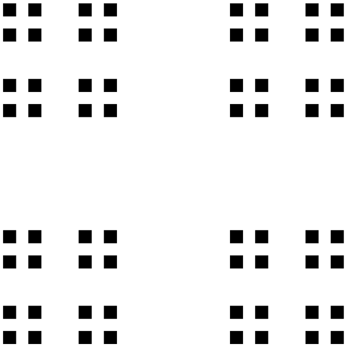
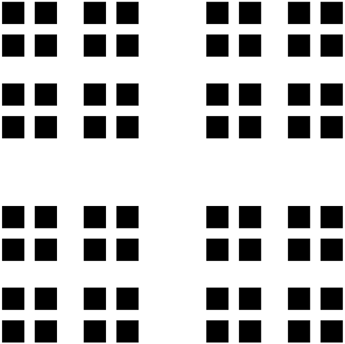
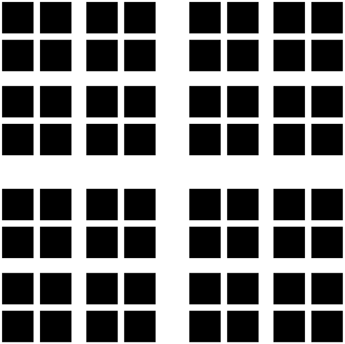
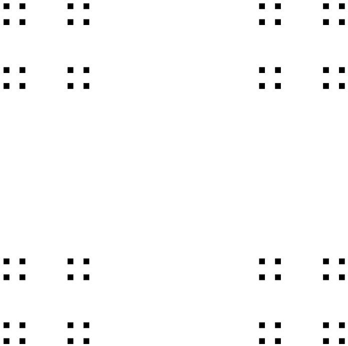
26.4.2012

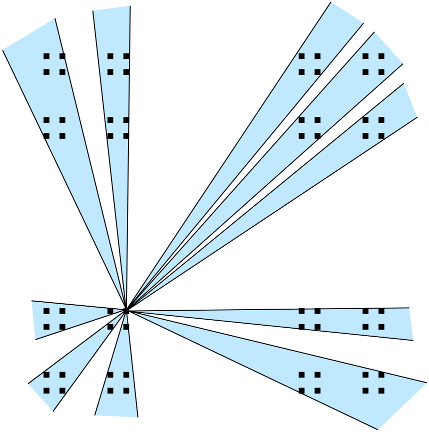
JOINT WORK WITH PABLO SHMERKIN AND VILLE SUOMALA

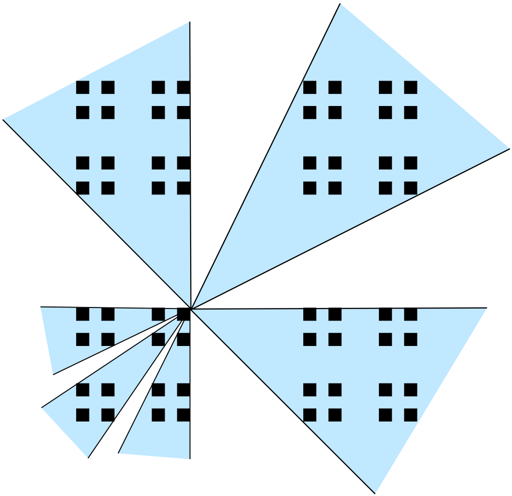


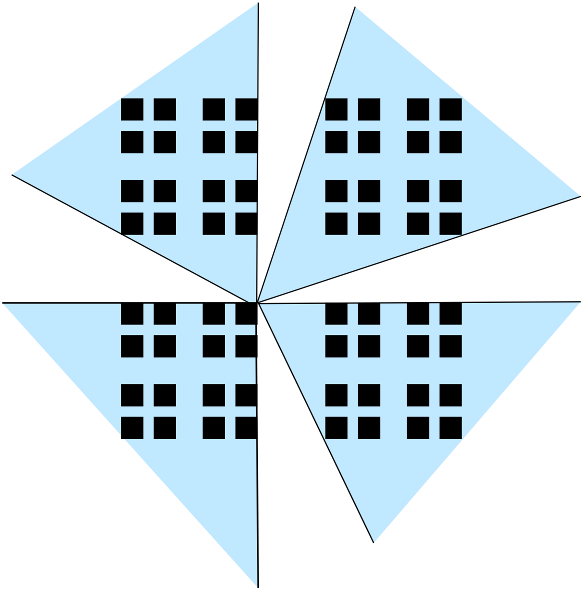


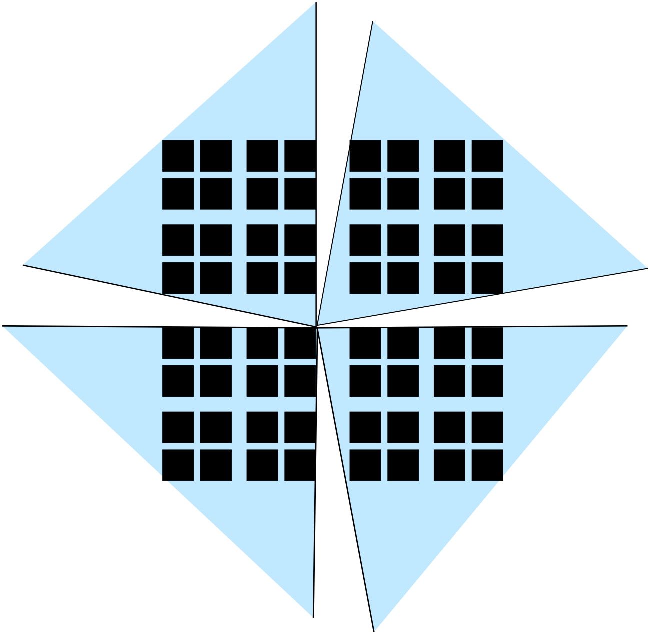












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How to make this precise?

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Geometry

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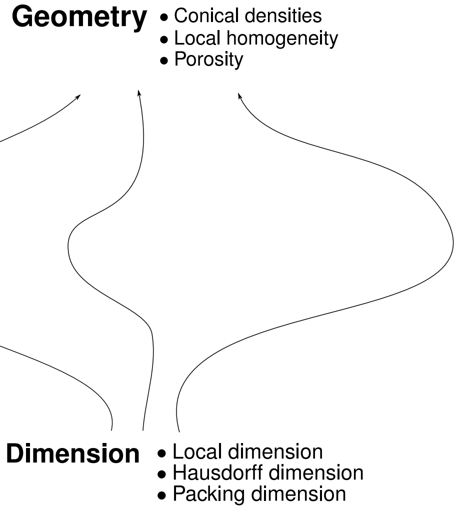
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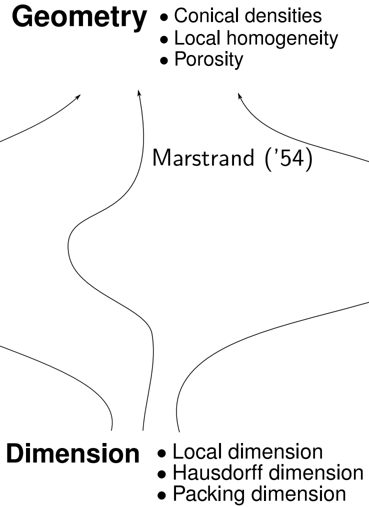
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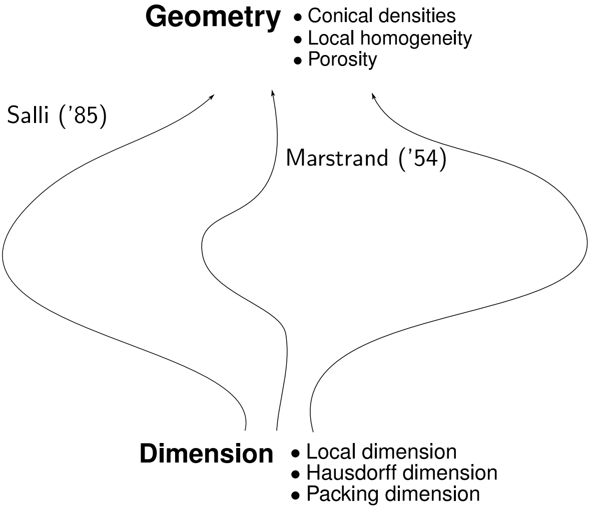

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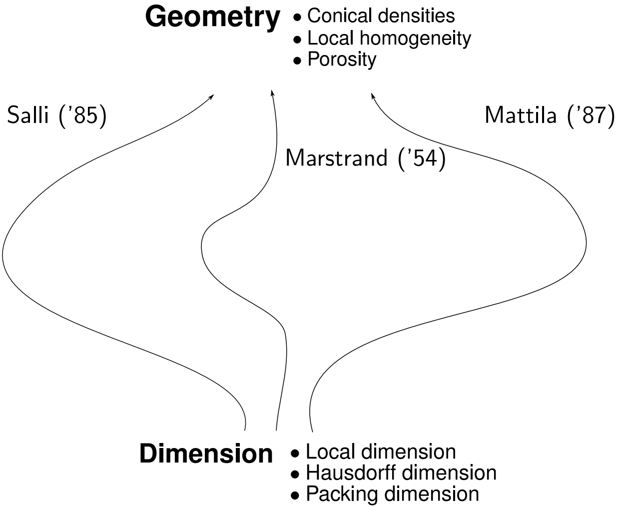
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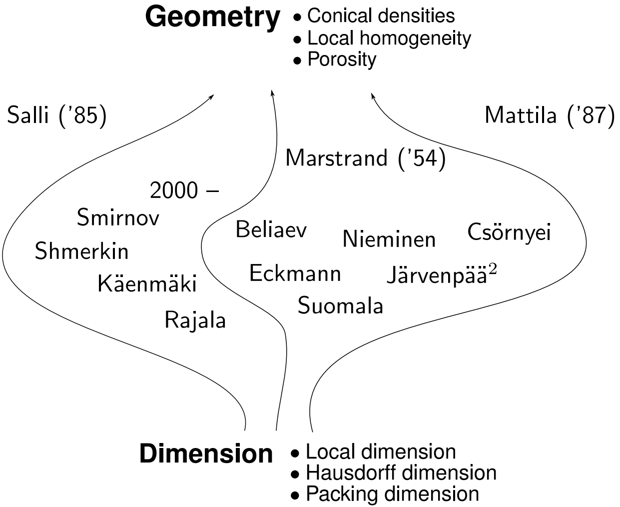

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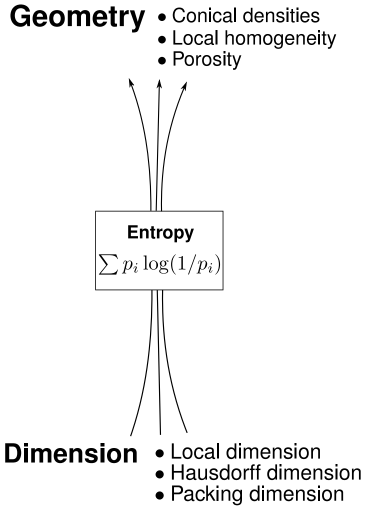
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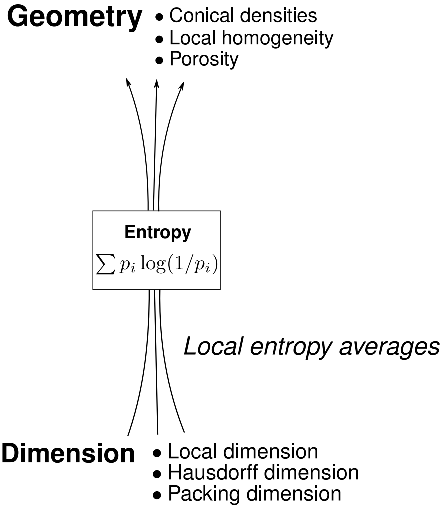
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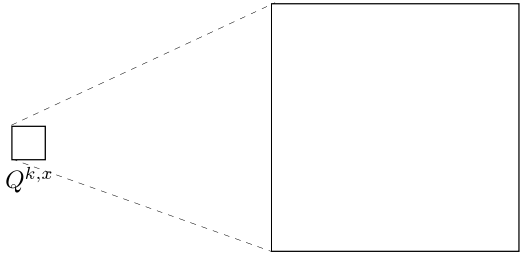

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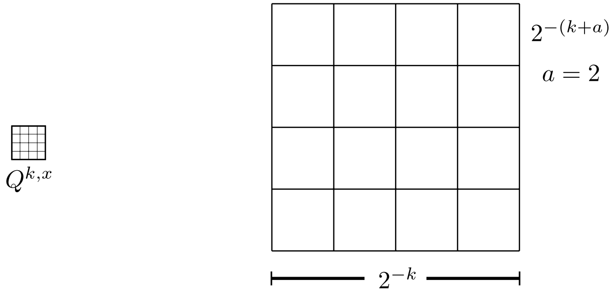

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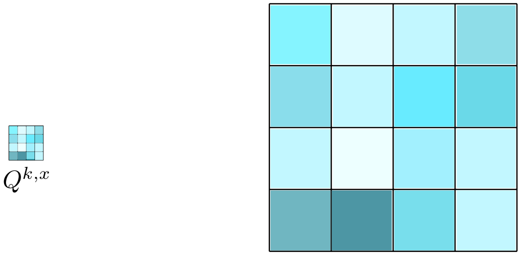

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$$\begin{aligned} \mathrm{por}_m(\mu, x, r, \varepsilon) = \sup\{\varrho > 0 : \; & \exists y_1, \ldots, y_m \in \mathbb{R}^d \text{ with} \\ & (y_i - x) \cdot (y_j - x) = 0, i \neq j, \\ & B(y_i, \varrho r) \subset B(x, r), \text{ and} \\ & \mu(B(y_i, \varrho r)) \leq \varepsilon \mu(B(x, r))\}. \end{aligned}$$

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$$\begin{aligned} \mathrm{por}_m(\mu, x, r, \varepsilon) = \sup\{\varrho > 0 : \ & \exists y_1, \ldots, y_m \in \mathbb{R}^d \text{ with} \\ & (y_i - x) \cdot (y_j - x) = 0, i \neq j, \\ & B(y_i, \varrho r) \subset B(x, r), \text{ and} \\ & \mu(B(y_i, \varrho r)) \leq \varepsilon \mu(B(x, r))\}. \end{aligned}$$

- μ is **lower mean** (m, α, p)**-porous** at x, if for any $\varepsilon > 0$ and for all large enough $N \in \mathbb{N}$ we have

  $$\mathrm{por}_m(\mu, x, r, \varepsilon) > \alpha$$

  for at least pN dyadic scales $r \in \{2^{-1}, 2^{-2}, \ldots, 2^{-N}\}$.

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- μ is **upper mean** (m, α, p)**-porous** at x if for all $\varepsilon > 0$, this holds for infinitely many $N \in \mathbb{N}$.

Theorem

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$$\overline{\dim}_{\text{loc}}(\mu, x) \leq d - pm + \frac{c(d)}{\log \frac{1}{1-2\alpha}},$$

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