

Applied Topology Bedlewo 2013

Book of Abstracts.

1.

HENRY ADAMS

Evasion paths in mobile sensor networks

Stanford University, USA

Suppose disk-shaped sensors wander in a planar domain. A sensor doesn't know its location but does know which sensors it overlaps. We say that an evasion path exists in this sensor network if a moving evader can avoid detection. Vin de Silva and Robert Ghrist give a necessary condition, depending only on the time-varying connectivity graph of the sensors, for an evasion path to exist. Can we sharpen this result? We consider an example where the existence of an evasion path depends not only on the network's connectivity data but also on its embedding. We also study the space of evasion paths using a generalization of the unstable J. F. Adams spectral sequence to diagrams of spaces.

2.

KARIM ALEXANDER ADIPRASITO

Minimal CW models for complements of 2-arrangements

Freie Universität Berlin, Berlin, Germany

A model for a topological space is a CW complex homotopy equivalent to it. In the best case, such models are chosen to be minimal, that is, they are chosen such that the number of i -cells of the model equals the i -th rational Betti number of the space. Unfortunately, not all spaces admit minimal models. In my talk, I will investigate the question whether complements of certain subspace arrangements admit minimal models. Previous work of Hattori, Dimca-Papadima, Randell and others answered this question positively for complex hyperplane arrangements. I will demonstrate a generalization of their results to the class of 2-arrangements introduced by Goresky and MacPherson. The main idea is to establish a Lefschetz-type hyperplane theorem for complements of 2-arrangements using discrete Morse theory of Forman and the theory of combinatorial stratifications of Björner and Ziegler.

3.

JAVIER ARSUAGA

Using computational homology to analyze breast cancer genomic data

San Francisco State University, San Francisco, CA

Genomic technologies measure thousands of molecular signals in order to understand essential biological processes. In cancer these molecular signals have been used to characterize disease subtypes, cancer pathways as well as subsets of patients with specific prognostic factors. This large amount of information however is so complex that new mathematical methods are called for. Computational homology provides one such method. We have developed a new homology based method to help identify clinically different subgroups. This method associates a set of point clouds to any given genomic profile and uses Betti numbers of the surfaces defined by the point cloud to detect differences between any two subgroups. When applied to gene expression data our method can distinguish patients with more aggressive tumors from those with less aggressive ones. When applied to copy number data we find regions of the genome that are associated with different tumor characteristics suggesting these regions contain genes that help tumors grow.

4.

PIOTR BARTŁOMIEJCZYK

The exponential law for partial, local and proper maps and its application to otopy theory.

Institute of Mathematics, University of Gdańsk, Poland

We introduce the topology in the set of local maps and prove the exponential law for partial, local and proper maps. We use these results to explain and clarify the basic relations between different spaces of partial maps in an Euclidean space: local, proper, local gradient and proper gradient.

This is joint work with Piotr Nowak-Przygodzki.

5.

PAVLE BLAGOJEVIĆ

On highly regular embeddings

Freie Universität Berlin, Germany

Given parameters k , ℓ , and d , we give new lower bounds on the dimensions N such that there are maps $\mathbb{R}^d \rightarrow \mathbb{R}^N$ that are k -regular, ℓ -skew, or (k, ℓ) -regular-skew. This extends and sharpens results due to Chisholm (1979) and Ghomi & Tabachnikov (2008).

This is joint work with Wolfgang Lück and Günter Ziegler.

6.

OMER BOBROWSKI
The Topology of Noise

Duke University

In manifold learning, one often wishes to infer geometric and topological features of an unknown manifold embedded in a d -dimensional Euclidean space, from a finite point cloud. One topological invariant of a considerable interest is the set of Betti numbers of a manifold. The zeroth Betti number stands for the number of connected components, whereas the higher Betti numbers represent the number of ‘holes’ of different dimensions. A standard way to recover this set of invariants is to study the union of d -dimensional balls with radius r , centered at the point cloud. This method, however, is highly sensitive to the choice of the radius r .

In this talk we will present limit theorems for the Betti numbers of such union of d -dimensional balls, as the number of points goes to infinity. The point clouds we study are generated from either a known density function in \mathbb{R}^d , or a probability measure supported on a compact submanifold. We show that different types of distributions and radius choices, lead to a variety of limit structures, with a completely different topological behavior. In particular, we present sufficient conditions under which the union of the balls successfully recovers the properties of the original manifold.

This is joint work with Robert Adler (Technion), Sayan Mukherjee (Duke), and Shmuel Weinberger (University of Chicago).

7.

PETER BUBENIK
Persistent homology of metric space valued functions

Department of Mathematics, Cleveland State University, USA

I will show how many flavors of persistent homology, including sublevelset, levelset, multidimensional, and angle-valued persistent homology can be understood using diagrams of ordered metric spaces. In this common framework, interleavings provide a metric, and functoriality implies stability. If the diagrams lie in an abelian category (e.g. persistence modules) then we have kernel, image and cokernel persistence and they too are stable.

Joint work with Jonathan A. Scott and Vin de Silva

8.

GIL CAVALCANTI
Formality Beyond Kähler geometry

Utrecht University, Utrecht, The Netherlands

Abstract: I will review the notion of formality for manifolds recall a result of Deligne, Griffiths, Morgan and Sullivan stating that Kähler manifolds are formal. Then I will move on to study the question "what else is formal?" Different extensions of the Kähler condition are complex and symplectic manifolds, generalized complex manifolds and manifolds with special holonomy and each of these extensions leads to something interesting.

9.

YOUNGGI CHOI
Module category weight of compact Lie groups

Seoul National University, Seoul, Korea

Every space X has a filtration given by the X -projective k -space $P^k(\Omega X)$ of its loop space ΩX . Let $e_m : P^m(\Omega X) \rightarrow P^\infty(\Omega X) \simeq X$ be the inclusion map. Then the module category weight $Mwgt(X; \mathbb{F}_p)$ is defined as the minimal number m such that $(e_m)^* : H^*(X; \mathbb{F}_p) \rightarrow H^*(P^m(\Omega X); \mathbb{F}_p)$ is a split monomorphism of modules over the Steenrod algebra, that is, there is a epimorphism $\phi_m : H^*(P^m(\Omega X); \mathbb{F}_p) \rightarrow H^*(X; \mathbb{F}_p)$ which preserves all Steenrod actions and $\phi_m \circ (e_m)^* \cong 1_{H^*(X; \mathbb{F}_p)}$.

In this talk, we compute the module category weight of some compact Lie groups including exceptional Lie groups.

10.

DANIEL C. COHEN
On the topology of matrix configuration spaces.

Dept. of Mathematics, Louisiana State University, Baton Rouge, LA, USA

We discuss some topological aspects of "matrix configuration spaces", certain generalizations of the classical configuration space of n distinct ordered points in the plane.

This is part of work in progress with Benson Farb.

11.

HELLEN COLMAN
Equivariant topological complexity

Wright College, Chicago, USA

We present an equivariant version of Farber's topological complexity for spaces with a given compact group action. This is a special case of the equivariant sectional category of an equivariant map. We show the relationship of these invariants with the equivariant Lusternik-Schnirelmann category. Several examples and computations serve to highlight the similarities and differences with the non-equivariant case. The equivariant topological complexity can be used to give estimates of the non-equivariant topological complexity.

This is joint work with Mark Grant.

12.

ARMINDO COSTA
Geometry and topology of random 2-complexes.

University of Warwick, United Kingdom

In this talk we will study random 2-dimensional complexes in the Linial-Meshulam model and prove that the fundamental group of a random 2-complex Y has cohomological dimension ≤ 2 if the probability parameter satisfies $p \ll n^{-3/5}$. Besides, for $n^{-3/5} \ll p \ll n^{-1/2-\epsilon}$ the fundamental group $\pi_1(Y)$ has elements of order two and hence has infinite cohomological dimension. Higher torsion will also be discussed if time allows. We will also describe a simple algorithmically testable criterion for a subcomplex of a random 2-complex to be aspherical; this implies that for $p \ll n^{-1/2-\epsilon}$ any aspherical subcomplex of a random 2-complex satisfies the Whitehead conjecture. This talk is based on joint work with M. Farber.

13.

JOÃO PITA COSTA
The Persistence Lattice.

Department of Artificial Intelligence (Jozef Štefan Institut, Ljubljana Slovenia)

The intrinsic connection between lattice theory and topology is fairly well established. For instance, the collection of open subsets of a topological subspace always forms a lattice. In this talk we will introduce an alternative interpretation of persistence based on the study of the order structure of its correspondent lattice.

Its algorithmic construction leads to two operations on homology groups which describe a diagram of spaces as a complete Heyting algebra, a generalization of a Boolean algebra. This reduces to known definitions of persistence in the cases of standard persistence and zigzag persistence. We will further discuss some of the properties of this lattice, the algorithmic implications of it, and possible applications.

This is a joint work with PRIMOŽ ŠKRABA (Jozef Štefan Institut, Ljubljana, Slovenia).

14.

LISBETH FAJSTRUP

Cut-off theorems in PV-models, a geometric approach.

Aalborg University, Aalborg, Denmark.

In Dijkstra's PV-model, a program is given by its use of shared resources. There is a set of resources r^i each with a limited capacity k_i . A *thread* is a list of requests for access P_i (if granted access, the thread locks the resource) and release of resources V_i . When several threads run in parallel, it may create conflicts once the capacity of some of the resources is reached. The geometric model of one thread is a graph representing loops and branches of the thread. The geometric model of a parallel program is the product of the graphs of the threads. Some points are cut out - hyper rectangles, where the capacity of a resource is superseded. An execution is a directed path from the joint initial state of all threads to the joint final state of all states. Here, we consider the special case, where a thread T is run in parallel with itself n times, given the joint execution T^n . A *cut-off theorem* is a result that a property holds for all n , if and only if it holds up to a fixed n . We give two such theorems. A deadlock is a state in T^n , where no thread can proceed, either because it reached its final state or because it requests access to a resource r^j which is locked by k_j other threads. The state where all threads have reached their final state is not a deadlock.

Theorem 0.1 *Given a thread T which accesses resources r^1, \dots, r^l of capacity k_1, \dots, k_l . Let T^n denote T in parallel with itself n times, then T^n is deadlock free for all n if and only if T^M is deadlock free, where $M = \sum_{j=1}^l k_j$, the sum of the capacities.*

A joint program T^n is serialisable if all execution paths are directed homotopy equivalent to a serial execution, executing one thread at a time.

Theorem 0.2 *Let T be a thread which accesses resources r^1, \dots, r^l , each of capacity $k = 1$. Then T^n is serialisable if and only if T^2 is serialisable.*

15.

ALEKSANDRA FRANČ

Some lower bounds for topological complexity.

Faculty of Computer and Information Science, University of Ljubljana, Slovenia

We introduce fibrewise Whitehead and Ganea definitions of monoidal topological complexity. We then define several lower bounds which improve on the standard lower bound in terms of nilpotency of the cohomology ring. Finally, we investigate spaces for which the topological complexity is close to the maximal possible value. We show that in these cases the gap between the lower bounds and the topological complexity is narrow and $\text{TC}(X)$ often coincides with the lower bounds.

This is joint work with Petar Pavešić.

16.

PETER FRANEK

Robust satisfiability of systems of equations

Institute of Computer Science, Academy of Sciences of the Czech Republic, Prague, Czech Republic

We describe the problem of robust satisfiability of general systems of n nonlinear equations in m variables. This is a problem to decide, whether for a compact m -manifold M^m , a continuous function $f : M^m \rightarrow \mathbb{R}^n$ and a number $\alpha > 0$, each $g : M^m \rightarrow \mathbb{R}^n$ such that $\|g - f\| < \alpha$, has a root in M^m or not. We show that this is closely connected to the *topological extension problem* for maps to the $(n - 1)$ -sphere.

The input data f and M^m are represented in an algorithmic way. We use recent results on algorithmic decidability of the extension problem to prove that robust satisfiability of systems of n equations in m variables is decidable if $m < 2n - 2$ and undecidable in general. In the case $m = n$, the problem reduces to an automatic computation of the topological degree $\deg(f, U, 0)$ for $U = \{|f| \leq \alpha\}$, but is more complicated for $n < m < 2n - 2$.

This is joint work with Marek Krčál.

17.

VIKTOR FROMM
Morse Homotopy and Homological Conformal Field Theory

Humboldt University Berlin

Flow graphs in a manifold M are continuous maps from a graph G to M , which satisfy along the edges of G prescribed flow equations. By studying these objects, invariants of manifolds can be recovered - the simplest instance of this is the classical Morse complex, corresponding to the case when the graph consists of a single edge. On the other hand, it is known from the work of Ralph Cohen et al. and Kenji Fukaya that suitably chosen graphs can also be used to encode for example the Steenrod Squares, the Stiefel-Whitney Classes or the Massey Products. These ideas lead to field-theoretic structures, i. e. the constructions are compatible with what happens when we glue two graphs together.

In the talk, I want to explain how to take into account an additional datum - a so-called ribbon structure on the graph - to construct what is roughly the conformal version of this theory. In fact, this turns out to be the geometric analogue of an algebraic-combinatorial construction that appeared some twenty years ago in the work of M. Kontsevich. By comparing the geometric and the algebraic approach, it can be explained what information about the manifold is recovered.

18.

MORITZ FIRSCHING
Equivariant models for open manifolds with an action of a finite group.

Freie Universität Berlin, Germany

When considering the existence of G -equivariant maps from an open manifold M into a test space for a finite group G , it is useful to find compact G -equivariant homotopy models for M . Open G -manifolds arise commonly when studying the complements of arrangements and configuration spaces of manifolds.

Motivated by problems in geometric combinatorics we study the existence of such models and show how they can be constructed in some cases. Moreover, we discuss a general theorem that guarantees, under certain conditions, the existence of equivariant maps without explicitly constructing models.

As a specific application we analyze the open $((\mathbb{Z}/2)^k \rtimes \mathfrak{S}_k)$ -manifold $F^{\mathbb{Z}/2}(S^n, k)$ of ordered k -tupels in S^n , that are pairwise distinct and pairwise antipodally distinct and show the existence of $((\mathbb{Z}/2)^k \rtimes \mathfrak{S}_k)$ -equivariant maps from $F^{\mathbb{Z}/2}(S^n, k)$ into certain $((\mathbb{Z}/2)^k \rtimes \mathfrak{S}_k)$ -equivariant spheres.

19.

JESUS GONZALEZ

Sequential motion planning of non-colliding particles in Euclidean spaces.

Cinvestav, Mexico.

Yuli Rudyak has proposed a natural extension of Farber's topological complexity, a concept related to the motion planning of an articulated system (robot). In Rudyak's view, the goal is to get information about the continuity instabilities of sequential motion planners of the robot. In this talk I will show how to extend the work of Farber, Grant and Yuzvinsky to compute Rudyak's higher topological complexity of configuration spaces of non-colliding ordered particles in a Euclidean space.

20.

MARK GRANT

Topological complexity of braid groups

University of Nottingham, Nottingham, UK

We present new lower bounds for the topological complexity of $K(G, 1)$ spaces which come from examining subgroups of G which intersect trivially. From another point of view, knowledge of the topological complexity of a $K(G, 1)$ can be used to show that certain subgroups of G must have non-trivial intersection.

We illustrate our lower bounds through examples given by braid groups.

This is joint work with Greg Lupton and John Oprea (Cleveland State).

21.

JACEK GULGOWSKI

Path following algorithm based on the sign changes

Institute of Mathematics, University of Gdańsk.

We are presenting a certain simplification of the well-known PL-continuation method, applied to continuous curve tracking, for zero set of the map $F : \mathbb{R}^{k+1} \rightarrow \mathbb{R}^k$. The presented algorithm builds a family of regular $k + 1$ -dimensional simplices following the set of zeros of the map F . The algorithm selects k -dimensional faces of the simplex showing non-zero value of the Brouwer degree of the map F restricted to the given face. Certain practical approximations of the topological conditions are presented. The algorithm is suggested, where the values of the map F are calculated in the vertices of each simplex, and – based on sign changes of coordinate functions of the map F – a decision is made as to which of the k -dimensional faces are to be taken as the faces intersected by the zero set $F^{-1}(0)$.

The presented path following algorithm requires no special procedures for bifurcation point detection and handling. It is shown the algorithm follows all paths also in the case of multiple bifurcations. The algorithm was successfully used before in the process of searching for the dispersion characteristics of the microwave transmission lines.

This is joint work with Jerzy Julian Michalski, TeleMobile Electronics, Gdynia, Poland

22.

SANG-EON HAN

A new approach to the study of digital spaces derived from a Khalimsky topological structure

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The talk deals with several new concepts such as an A -map, an A -isomorphism, an A -retract and so forth which are related to a Khalimsky topological structure. Comparing with a Khalimsky continuous map, a Khalimsky homeomorphism and a Khalimsky retract, we show that these new notions can be very flexible and efficient for the process of compressing Khalimsky topological spaces so that this approach can contribute to computer science such as image analysis, image processing, computer graphics, mathematical morphology and so forth.

23.

MICHELE INTERMONT

Some Results in Visualizing Data

Kalamazoo College, Kalamazoo, MI, USA

In this talk we examine some fMRI data in an attempt to understand which portions of the brain are activated by specific stimuli. The topological tool we use is Mapper, introduced by Singh, Memoli and Carlsson in 2007. We also discuss some stability results for Mapper.

24.

KOUYEMON IRIYE

Wedge decomposition of polyhedral products

Osaka Prefecture University, Sakai, Japan

In 1999, Herzog, Reiner and Welker show that the Alexander dual of a sequentially Cohen-Macaulay complex is Golod, that is, the multiplication and higher Massey products in the torsion algebra associated with the Stanley-Reisner ring are trivial. We will show this result by studying the homotopy type of the moment-angle complex. In fact, we decompose a polyhedral product associated with a simplicial complex with the sequentially Cohen-Macaulay Alexander dual as a wedge sum of suspension spaces, where a polyhedral product is a homotopy theoretical generalization of a moment-angle complex.

This talk is based on joint works with Daisuke Kishimoto (Kyoto Univ.).

25.

GRZEGORZ JABŁOŃSKI
Persistent homology of maps
Uniwersytet Jagielloński, Kraków, Poland

Persistent homology is used to analyze the data from sampled topological spaces. I would like to present how one can use persistent homology to extract information about a self-map acting on topological space. As an input we have only sampled space and information how the map acts on the sample. General idea of our method is based on applying persistent homology to eigenspaces of the map induced by the given function in homology. This is joint work with Herbert Edelsbrunner and Marian Mrozek.

26.

DUŠKO JOJIĆ
Shellability of complexes of directed trees.
Faculty of Science, University of Banja Luka
78 000 Banja Luka, Bosnia and Herzegovina

Establishing shellability of a simplicial complex is an easy combinatorial way to obtain a lot of information about the topology of this complex. The question of shellability of complexes of directed trees was asked by R. Stanley. D. Kozlov showed that the existence of a complete source in a directed graph provides a shelling of its complex of directed trees. We will show that this property gives a shelling that is straightforward in some sense. Among the simplicial polytopes, only the crosspolytopes allow such a shelling. Furthermore, we show that the complex of directed trees of a complete double directed graph is a union of iterated bipyramids over the boundary of simplex. We prove that the complex of directed trees of a directed graph which is essentially a tree is vertex-decomposable. For these complexes we describe their sets of generating facets. Also, we find a relation between homology of these complexes and some combinatorial invariants for trees.

27.

SHIZUO KAJI
An Application of Lie theory to Computer Graphics
Yamaguchi university, Yamaguchi, Japan

In computer graphics, various mathematics is used such as the Navie-Stokes equations for generating clouds and fires, reproducing kernel Hilbert space for mixing different facial expressions, and piecewise linear map (PL-map, in short) for morphing shapes. I will discuss an algorithm to blend/deform shapes based on PL-map and an elementary Lie theory. A shape is represented by a polyhedron and its deformation by a PL-map. The idea is to find a suitable PL-map which minimizes a certain energy functional defined on the space of 3-dimensional PL-maps.

ROMAN KARASEV

An analogue of Gromov's waist theorem for coloring the cube.

Dept. of Mathematics, Moscow Institute of Physics and Technology, Institutskiy per. 9,
Dolgoprudny, Russia 141700

One possible way to express that the cube $Q^d = [0, 1]^d$ has dimension d is to notice that it cannot be colored in d colors with arbitrarily small connected monochromatic components. A more precise statement was proved by Lebesgue: If the unit cube Q^d is covered by closed sets with multiplicity at most d then one covering set must meet two opposite facets of Q^d .

Matoušek and Přívětivý (2008) asked a similar question in combinatorial setting:

Question. If we color Q^d in $m + 1$ colors (to make the problem discrete we color small cubes of the partition of Q^d into n^d small cubes) then what size of a monochromatic connected component can we guarantee?

The corresponding combinatorial statement for $m = d - 1$, corresponding to Lebesgue's theorem, is called the HEX lemma. In this case there must exist a monochromatic connected component spanning two opposite facets and such a component must consist of at least n small cubes.

Matoušek and Přívětivý considered colorings in 2 colors using isoperimetric inequalities for the grid and a lower bound $n^{d-1} - d^2 n^{d-2}$ for the size of a connected monochromatic component was established. They also conjectured that the size of a monochromatic connected component is of order n^{d-m} for $m + 1$ colors when d and m are fixed.

Alexey Kanel-Belov also posed the same problem in 1990s (private communication) and it circulated among mathematicians in Moscow and was posed at some olympiad-like events, e.g. the Tournament of Towns 2010. As a result, a different proof of this result was obtained by an undergraduate student Marsel Matdinov, see arXiv:1111.3911.

29. JAREK KEDRA

On the L^2 -geometry of volume preserving diffeomorphisms.

University of Aberdeen and University of Szczecin

This is joint work with Michael Brandenbursky

Hydrodynamics can be modelled by the L^2 -geometry on the group of volume preserving diffeomorphisms of the configuration space. According to a theorem by Schirelmann, the diameter of such diffeomorphism group of a compact simply connected region in \mathbf{R}^3 equipped with the L^2 -metric is bounded. I will prove a theorem stating that if the topology of the configuration space is complicated enough then its group of volume preserving diffeomorphisms equipped with the L^2 -metric contains quasiisometrically embedded free abelian subgroup of arbitrary rank or a direct product of nonabelian free groups. In particular, it is unbounded.

30.

DAISUKE KISHIMOTO
Hom-complexes and hypergraph colorings.
Kyoto University Department of Mathematics, Kyoto, Japan

We generalize Hom-complexes of graphs to uniform hypergraphs (with multiplicities) and give topological lower bounds for their chromatic numbers following Babson and Kozlov's programme. We also discuss the hierarchy of known topological lower bounds for the chromatic numbers of uniform hypergraphs through Hom-complexes.

31.

RAFAŁ KOMENDARCZYK
Knot and link invariants for vector fields.
Tulane University, New Orleans, United States

I will review the Arnold's construction of the asymptotic linking number for a divergence free vector field, and indicate its possible extensions to other invariants of knots and links.

32.

ULRICH KOSCHORKE
Nielsen coincidence numbers, Hopf invariants and spherical space forms
Universitaet Siegen, Germany

Given two maps between smooth manifolds, we want to study their coincidence set (where their values agree - a fixed point set is an important special case). The obstruction to removing such coincidences by deforming the maps is measured by minimum numbers (of coincidence points or components). In order to determine them we introduce and study an infinite hierarchy of Nielsen numbers N_i , $i = 0, 1, \dots$. They approximate the minimum numbers from below with decreasing accuracy, but they are more easily computable as i grows. If the domain and the target manifold have the same dimension (e.g. in the fixed point setting) all these Nielsen numbers agree with the classical definition. However, in general they can be quite distinct.

While our approach is very geometric the computations use the techniques of homotopy theory and, in particular, all versions of Hopf invariants (a la Ganea, Hilton, James...). As an illustration we determine all Nielsen numbers and minimum numbers for pairs of maps from spheres to spherical space forms. Maps into even dimensional real projective spaces turn out to produce particularly interesting coincidence phenomena (involving e.g. also Kervaire invariants).

33.

SANJEEVI KRISHNAN
Higher dimensional flow-cut dualities.

University of Pennsylvania.

Some optimization dualities, such as the max-flow min-cut theorem, are trivial cases of a Poincare Duality for (co)homology on sheaves of semimodules. This talk will present the theorem, give examples of other flow-cut dualities (e.g. smooth, higher-dimensional, monoid-theoretic) that arise from the theorem, and work out some relevant and illuminating calculations. No familiarity with semimodule theory will be assumed.

34.

VITALIY KURLIN
Reconstructing persistent structures from noisy images

Durham University, Durham, United Kingdom

Let a point cloud be a noisy dotted image of a graph with straight edges in the plane. We present a new algorithm for reconstructing the original graph from the given point cloud without any user-defined parameters. We locally analyze each point p in the given cloud to determine whether p should be near a vertex or an edge in a reconstructed graph. Then we split the cloud into clusters of vertex-points and edge-points. All parameters are automatically tuned by using persistent homology. We give conditions on an unknown graph G and a given cloud around G when the algorithm correctly reconstructs the topological type of G . A 4-page description of the algorithm and an early version of the Java applet for reconstructing graphs are at <http://www.maths.dur.ac.uk/~dma0vk/java-applets/graph-reconstruction.html>.

This joint work with Dr Alexey Chernov (University of Bedfordshire, Luton and Computer Learning Research Centre, RHUL, Egham, UK) was supported by the EPSRC grant *Persistent topological structures in noisy images* EP/I030328/1.

35.

RAN LEVI
The topology of neural systems.

University of Aberdeen, UK.

This talk is a report on an ongoing project that aims to contribute to the understanding of certain aspects of brain activity. The project is carried out jointly with Kathryn Hess, and Sophie Raynor, in collaboration with the Blue Brain Project in EPFL, and as a part of the Human Brain Project.

The brain, or possibly any part of it, can be viewed as a graph in several ways, the simplest of which is by regarding the neurons as vertices with an edge between two vertices if they are connected to each other by a synapse. One naturally regards this graph as oriented, as the connections in a neural system are directional, but for some purposes ignoring orientation is also interesting. Experimental data will impose restrictions on the nature of such a graph, for instance on the number and strength of connections between any given vertex and other vertices. Once a graphical description is the basic concept one deals with, a variety of mathematical structures that may be biologically relevant emerge. In this talk I will discuss a number of ideas that we have been investigating, ranging from the application of basic techniques of algebraic topology to the study of state of the art simulated neural systems, to a category theoretic model of neural systems which encodes not only their structure, but also the activity within them and their ever changing states.

36.

WOJCIECH LUBAWSKI

A new approach to the equivariant topological complexity

Theoretical Computer Science, Jagiellonian University

We will present a new approach to equivariant version of the topological complexity called a symmetric topological complexity. It seems that the presented approach is more adequate for the analysis of impact of symmetry on the motion planning algorithm than the one introduced and studied by Colman and Grant. We will show many bounds for the symmetric topological complexity comparing it with already known invariants and prove that in the case of a free action it is equal to the Farber's topological complexity of the orbit space. We will define the Whitehead version of it.

This is joint work with Waclaw Marzantowicz

37.

STEFANO MARÒ

Periodic solutions with winding number N of a forced relativistic pendulum.

Università di Torino - Dipartimento di Matematica, Italy

We prove the existence of at least two geometrically different periodic solutions with winding number N for the equation of the forced relativistic pendulum. These solutions are such that $x(t + T) = x(t) + 2N\pi$ with N satisfying a physically necessary condition. The instability of a solution is also proved. The proof is topological and based on the version of the Poincaré-Birkhoff theorem by Franks.

38.

FACUNDO MÉMOLI
Curvature sets over Persistence Diagrams.
 University of Adelaide

A combinatorial idea of Gromov is to assign to each metric space the collection $\{K_n(X)\}_{n \in \mathbb{N}}$ of sets each consisting of all distance matrices corresponding to all possible n -tuples of points in X . It is known that $K_n(\cdot)$ is 2-Lipschitz w.r.t. the Gromov-Hausdorff distance. The proposed extension is: given a filtration the functor \mathcal{F} on finite metric spaces consider $K_n^{\mathcal{F}}(X)$, the *set of all possible \mathcal{F} -persistence diagrams generated by metric subsets of X of cardinality n* . Is there a sense in which $K_n^{\mathcal{F}}$ is stable? For a class of filtration functors which we call *compatible*, the answer is positive, and these admit stability results in the Gromov-Hausdorff sense.

In order to capture frequency or statistics, it is more useful to consider that, in addition to a metric structure, a probability measure has been specified. Then, to an mm-space X one assigns the collection $\{U_n(X)\}_{n \in \mathbb{N}}$ of all measures each defined on $\mathbb{R}^n \times \mathbb{R}^n$ each and given by the pushforward of the n -fold product measure $\mu_X^{\otimes n}$ under the map $\Phi_n : X^{\times n} \rightarrow \mathbb{R}^n \times \mathbb{R}^n$ which sends (x_1, x_2, \dots, x_n) to the distance matrix $(d_X(x_i, x_j))_{i,j=1}^n$. The link is that $\text{supp}(U_n(X)) = K_n(X)$.

This construction can be adapted to give a *measured version of $K_n^{\mathcal{F}}$* which can encode the statistics of persistence diagrams arising according to a given filtration functor \mathcal{F} . The proposal is to consider the pushforward measure $U_n^{\mathcal{F}}(X) := (D_* \circ \mathcal{F} \circ \Phi_n)_{\#} \mu_X^{\otimes n}$ induced on barcode space. This measured version is obviously connected with the combinatorial construction: $\text{supp}(U_n^{\mathcal{F}}(X)) = K_n(X)$. The stability of these constructions can now be expressed in Gromov-Wasserstein sense.

39.

ROY MESHULAM
Random Latin Squares and 2-Dimensional Expanders
 Technion, Haifa, Israel

Let X be a 2-dimensional simplicial complex. The degree of an edge e is the number of 2-faces of X containing e . The complex X is an ϵ -expander if the coboundary $d_1\phi$ of every \mathbb{F}_2 -valued 1-cochain $\phi \in C^1(X; \mathbb{F}_2)$ satisfies

$$|\text{supp}(d_1\phi)| \geq \epsilon \min\{|\text{supp}(\phi + d_0\psi)| : \psi \in C^0(X; \mathbb{F}_2)\}.$$

We show the existence of 2-dimensional ϵ -expanders with maximum edge degree d for some fixed $\epsilon > 0$ and d . This is done via the following new model of random 2-dimensional complexes. A Latin square of order n is an n -tuple $L = (\pi_1, \dots, \pi_n)$ of permutations on $\{1, \dots, n\}$ such that $\pi_i^{-1}\pi_j$ is fixed point free for $1 \leq i < j \leq n$. Let $\{a_i\}_{i=1}^n, \{b_i\}_{i=1}^n, \{c_i\}_{i=1}^n$ be three disjoint sets and let (L_1, \dots, L_d) be a d -tuple of independently chosen random Latin squares of order n . For $1 \leq k \leq d$, let $L_k = (\pi_{k1}, \dots, \pi_{kn})$. Let $Y(L_1, \dots, L_d)$ be the 3-partite 2-dimensional complex consisting of all 2-simplices $[a_i, b_j, c_{\pi_{ki}(j)}]$ where $1 \leq i, j \leq n$ and $1 \leq k \leq d$.

It is shown that there exist $d < \infty$ and $\epsilon > 0$ such that the complex $Y(L_1, \dots, L_d)$ is an ϵ -expander with probability tending to 1 as $n \rightarrow \infty$. Joint work with Alex Lubotzky.

40.

WASHINGTON MIO
**Multiscale Analysis of Data Through Tensor Fields Underlying
Their Distribution**

Florida State University, Tallahassee, USA

We describe a multiscale approach to the construction of statistical summaries of data sets in Euclidean space employing a method that also provides a gateway to the geometry and topology of the data distribution. Basic statistics such as the mean and the covariance tensor yield widely used data summaries such as principal component analysis. However, these methods tend to be ineffective, for example, if the distribution is multimodal or the underlying geometry is fairly complex. The proposed multiscale counterparts capture more structure and offer additional insights on data organization.

The attractors of the (negative) gradient fields of scale dependent analogues of the Fréchet sum-of-squares function provide a generalized notion of “mean” and their stable manifolds may be interpreted as data clusters detectable at different scales. Bifurcation patterns reveal structures associated with the data organization that persist across scales. Similarly, multiscale covariance fields are used to quantify variation, not just with respect to the mean, but all points in the data landscape.

We also provide evidence that, for data supported on a submanifold M of Euclidean space, the curvature tensor and several global topological properties of M can be recovered from small scale covariance. In particular, we obtain a Gauss-Bonnet theorem for surfaces in \mathbb{R}^3 stated purely in terms of covariance fields and topology.

This is joint work with Diego Diaz Martinez and Facundo Mémoli.

41.

MONICA NICOLAU

Tackling the topology and geometry underlying big data

Stanford University, Stanford, USA

The recent onslaught of data has brought about profound changes in understanding a range of phenomena as dynamic, high complexity processes. New technology has provided an unprecedented wealth of information, but it has generated data that is hard to analyze mathematically, thereby making its interpretation difficult. These challenges have given rise to a myriad novel exciting mathematical problems and have provided an impetus to modify and adapt traditional mathematics tools, as well as develop novel techniques to tackle the data analysis problems. I will discuss a general approach to address some of these computational challenges by way of a combination of geometric data transformations and topological methods. In essence geometric transformations deform the data to focus intensity on a range of relevant questions, and topological methods identify statistically significant shape characteristics of the data. These methods have been applied in a wide range of settings, in particular for the study of the biology of disease. I will discuss some concrete applications of these methods, including their use to discover a new type of breast cancer, identify disease progression trends, and highlight the driving mechanisms in acute myeloid leukemia. While the specifics of the work are focused on biological data analysis, the general approach addresses computational challenges in the analysis of any type of large data.

42.

RAFAEL ORTEGA

Some dynamical properties of analytic diffeomorphisms of the plane

Universidad de Granada, Granada, Spain

Many dynamical properties of general planar homeomorphisms can be made more precise in the class of real analytic diffeomorphisms. In this talk I plan to review some results in this line. In particular I will discuss the dynamics of stable fixed points for maps that are area-preserving and orientation reversing.

43.

PETAR PAVEŠIĆ

Change-of-fibre for fibrewise-pointed spaces

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

The topological complexity introduced by M. Farber and the higher topological complexity as defined by Y. Rudyak may be both viewed as instances of the fibrewise Lusternik-Schnirelmann category of suitably chosen fibrewise-pointed spaces. In order to extend methods from the classical LS-category theory to the fibrewise setting we will introduce a construction that for a given continuous endofunctor on the category of pointed topological spaces yields a corresponding operation on the fibres of fibrewise-pointed spaces. In this way we will be able to construct the Whitehead-Ganea framework for the topological complexity.

44.

JOSE PEREA

Persistent Homology of Time-Delay Embeddings

Mathematics Department, Duke University, Durham NC, USA

We present in this talk a theoretical framework for studying the persistent homology of point clouds from time-delay (or sliding window) embeddings

$$SW_{M,\tau}f(t) = \begin{bmatrix} f(t) \\ f(t + \tau) \\ \vdots \\ f(t + M\tau) \end{bmatrix}$$

Here $f : \mathbb{R} \rightarrow \mathbb{R}$, $M \in \mathbb{N}$, $\tau > 0$ and $t \in \mathbb{R}$. In particular, we propose maximum persistence as a measure of periodicity at the signal level, f , present structural theorems for the resulting diagrams, and derive estimates for their dependency on the window size $M\tau$ and embedding dimension $M + 1$. We apply this methodology to quantifying periodicity in synthetic signals, and present comparisons with state-of-the-art methods in gene expression analysis.

This is joint work with John Harer at Duke University, Mathematics Department.

45.

MARTIN PINSONNAULT
Homotopy Type of Symplectomorphism Groups

The University of Western Ontario, London, Canada

Since the seminal work of Gromov, the rational homotopy type of symplectomorphism groups of closed 4-manifolds have been a subject of much research. In this talk, I will briefly survey the main techniques employed to investigate these infinite dimensional topological groups. I will also present examples of symplectomorphism groups whose rational homotopy algebra are infinite dimensional, but whose Pontryagin rings are finitely generated.

46.

MICHAEL ROBINSON
Morphisms between logic circuits.

American University, Washington, DC, USA

Designers of logic circuits usually work hierarchically: complex circuits are built of simpler ones, whose internal details have been abstracted away. For instance, two circuits whose truth tables are equivalent are often treated as having the same behavior. While this idea works in traditional synchronous systems, the temporal behavior of two implementations of the same boolean function may be quite different. These circuits can still be manipulated using a hierarchy of abstractions, but there is not a unified theory for their design. Recently, an intermediate family of algebraic logic circuit invariants have been discovered that arise from the theory of constructible sheaves on graphs. This talk will outline a way to analyze a circuit at different levels of detail, by describing sheaf morphisms between connection diagrams and their associated induced maps.

47.

KALLEL SADOK
On the Topology of Diagonal Arrangements and their Complements.

American University Sharjah and USTL, Lille

We start a thorough study of the so-called diagonal arrangements in X^n , where X is any reasonable topological space. In particular we consider the fat diagonals (ordered and unordered versions). Their complements in X^n generalize the more familiar configuration space of distinct points. We give an expression of their homotopy and homology groups through a range in terms of those of X .

48.

J. J. SÁNCHEZ–GABITES

Čech cohomology of attractors of discrete dynamical systems

Departamento de Anlisis Econmico. Universidad Autnoma de Madrid. Spain.

Suppose K is an asymptotically stable attractor for a dynamical system on \mathbb{R}^n . When the dynamical system is continuous (a flow) it is well known that the inclusion of K in its basin of attraction $\mathcal{A}(K)$ induces isomorphisms in Čech cohomology. In this talk we discuss whether the same holds true when the dynamical system is discrete (a homeomorphism). We show that (i) it is true if coefficients are taken in \mathbb{Q} or \mathbb{Z}_p (p prime) and (ii) it is true for integral cohomology if and only if the Čech cohomology of K or $\mathcal{A}(K)$ is finitely generated. We compute the Čech cohomology of periodic point free attractors of volume-contracting \mathbb{R}^3 -homeomorphisms.

This a joint paper with Francisco R. Ruiz del Portal, Departamento de Geometra y Topologa, Universidad Complutense de Madrid.

49.

PARAMESWARAN SANKARAN

Formality of Schubert varieties in classical complex flag manifolds.

The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600113, India

Suppose that $Y = X \cup_{\alpha} e^n$ is a cell-attachment where X is a CW complex which is formal. We obtain a criterion for the formality of Y . As an application we show that any union of Schubert varieties in a generalized complex flag variety G/B , where G is a complex semisimple Lie group and B , a Borel subgroup is formal. The same result is also obtained for Schubert ‘varieties’ in quaternionic complete flag manifolds. Also we obtain a new proof of a result of Panov and Ray that any torus manifold over a homology polytope where the torus action is locally standard action is formal.

This is based on joint work with Prateep Chakraborty.

50.

EDIVALDO L. DOS SANTOS

Borsuk-Ulam theorems and their parametrized versions for spaces of type (a, b) .

Federal University of São Carlos, São Carlos, Brazil.

Let X be a simply connected finite CW complex with \mathbb{Z} -cohomology groups satisfying $H^j(X; \mathbb{Z}) = \mathbb{Z}$, if $j = 0, n, 2n$ or $3n$, and $H^j(X; \mathbb{Z}) = 0$, otherwise ($n > 1$). Let u_i generate $H^{in}(X; \mathbb{Z})$, for $i = 0, 1, 2$ and 3 . Then the structure of the \mathbb{Z} -cohomology ring of X is determined by the two integers a and b for which $u_1^2 = au_2$ and $u_1u_2 = bu_3$. In this case, X is said to be of type (a, b) . Let X be a space of type (a, b) equipped with a free G -action, with $G = \mathbb{Z}_2$ or S^1 . In this work, we study some Borsuk-Ulam problems and the corresponding parametrized versions for such G -spaces.

This is joint work with Denise de Mattos and Pedro Pergher.

51.

DIRK SCHÜTZ
Intersection Homology of Linkage spaces.

Durham University, UK

We consider the moduli spaces $\mathcal{M}_d(\ell)$ of a closed linkage with n links and prescribed lengths $\ell \in \mathbb{R}^n$ in d -dimensional Euclidean space. For $d > 3$ these spaces are no longer manifolds generically, but they have the structure of a pseudomanifold.

We use intersection homology to assign a ring to these spaces that can be used to distinguish the homeomorphism types of $\mathcal{M}_d(\ell)$ for a large class of length vectors in the case of d even. This result is a high-dimensional analogue of the Walker conjecture which was proven by Farber, Hausmann and the author.

52.

NICHOLAS SCOVILLE
Discrete Lusternik–Schnirelmann category

Ursinus College, USA

The discrete version of Morse theory due to Robin Forman is a powerful tool utilized in the study of topology, combinatorics, and mathematics involving the overlap of these fields. Inspired by the success of discrete Morse theory, we take the first steps in defining a discrete version of the Lusternik–Schnirelmann category suitable for simplicial complexes. This invariant is based on collapsibility as opposed to contractibility. We will discuss some basic results of this theory, showing where it agrees and differs from that of the smooth case, as well as computational estimates of the discrete LS category. In addition, we will discuss a discrete version of the Lusternik–Schnirelmann theorem relating the number of critical points of a discrete Morse function to its discrete category.

53.

EVGENY SHCHEPIN
A Persistent Morse Theory in Plane.
Delone Laboratory of Discrete and Computational Geometry,

Let D be a bounded connected domain in the plane bounded by a number of disjoint closed broken lines. Let us fix a right line L which does not intersect D and is in general position with respect to its boundary ∂D . A vertex $A \in \partial D$ is called *L-critical* if the distance from A to L takes in A locally minimal or maximal value.

For any L -critical point one defines its *persistence* in the following way: Denote by L_A the line which is parallel to L and pass through A . And denote by $\partial D(A)$ the component of the boundary ∂D containing A . If $\partial D(A) \cap L_A$ does not contain points different from A the L -persistence of A is declared to be infinite.

If $\partial D(A) \cap L_A$ contains more than one point, then (by the general position) it contains at least three points. And there are two uniquely defined points $A', A'' \in \partial D(A) \cap L_A$ such that the arcs AA' and AA'' of the curve $\partial D(A)$ intersect L_A just in its endpoints. Now we can define the L -persistence of A as the minimal ε , such that the closed ε -neighborhood of L_A contains at least one of the arcs AA' or AA'' .

Theorem. For a given bounded PL-domain D , line L and $\varepsilon > 0$ there exists an orientation preserving piece-wise linear homeomorphism h of the plane such that:

- (a) $h(A) = A$ for all L -critical points of D with L -persistence more than ε
- (b) the images of points which L -persistence is more than ε are only L -critical points of $h(D)$
- (c) for any point x of the plane one has $|\text{dist}(x, L) - \text{dist}(h(x), L)| < \varepsilon$

54.

VIN DE SILVA

Persistent cohomology and the topological analysis of recurrent signals

Pomona College, USA

I will present a protocol for studying the recurrence properties of time-series data, by constructing auxiliary coordinates on the signal that reveal its topological properties. These coordinates take values in the circle, rather than the real line. For instance, we discover the period of a periodic signal without any kind of Fourier analysis. The method makes it easy to discover, heuristically, the quasiperiodic behaviour of chaotic systems such as the one containing the Lorenz attractor. The main tools are Takens delay embedding, persistent cohomology, and discrete Hodge theory. This is joint work with Primoz Skraba and Mikael Vejdemo-Johansson, with contributions by Dmitriy Morozov and Konstantin Mischaikow.

STANISŁAW SPIEŻ

Borsuk-Ulam type theorems and equilibria in a class of games.

IM Pan, Warsaw.

In 1968, R. Aumann, M. Mashler and R. Sterns posed a problem, whether any undiscounted infinitely repeated two-person game of incomplete information on one side has a Nash equilibrium. A brief description of these games is as follows. A game between two players \mathcal{A} and \mathcal{B} proceeds in infinitely many successive stages. In the 0-stage a k is chosen from a finite set K of "states of nature" according to a probability distribution known to both players. In any subsequent stage each of the players selects a "pure action" from a finite sets I (for \mathcal{A}) and J (for \mathcal{B}), gaining a stage-payoff $A_k(i, j)$ (for \mathcal{A}) or $B_k(i, j)$ (for \mathcal{B}), which depends only on the pure actions $i \in I$ and $j \in J$ selected in this stage and the "true state of nature" k , chosen at stage 0. At any stage the players also know the pure actions both of them took on proceedings stages and \mathcal{A} (but not \mathcal{B}) knows the outcome $k \in K$ of the 0-stage. We settle in the positive the problem stated above and extend this result to more general games of this type.

Several classical proofs in game theory depend on various fixed point and related theorems. Our proofs depend on new topological results. One of them, in its simplest form, states that if x_0 is a point of a compact subset C of R^n and $f : C \rightarrow Y$ is a mapping such that dimension of $f(\text{int}C)$ is less than n , then in the boundary of C there exists a set C_0 mapped by f into a singleton and containing x_0 in its convex hull. The resemblance with Borsuk-Ulam theorem is that if C is an n -ball and $Y = R^{n-1}$, then the later says that C_0 may be taken so as to consist of two points only. We also prove a parametric version of Borsuk-Ulam theorem which solves a problem related to a conjecture that is relevant for the construction of equilibrium strategies in a very general class of repeated two-player games with incomplete information.

This research is joint with T. Schick, R. S. Simon and H. Toruńczyk.

ANDRÁS SZÚCS

Homologies are infinitely complex.

Etvös Lornd University, Hungary.

It is well known that not all Z_2 -homology classes of manifolds can be realized by embedded submanifolds. We consider the question whether they can be realized by stratified subsets? Of course the class of all stratified subsets is sufficient to realize any homology class in any manifold. But if we admit only a finite set of "normal structures" of the strata then this is never enough in any codimension greater than 1. This means that for any fixed finite set θ of normal structures of the strata (which needs to be defined, but its definition is quite natural) there is a manifold (of sufficiently high dimension) and a homology class in it of codimension greater than 1 that can not be realized by any stratified subset having normal structures from the set θ .

57.

DAI TAMAKI

Some applications of cellular stratified spaces.

Department of Mathematical Sciences, Shinshu University, Matsumoto, Japan

The notion of cellular stratified spaces was introduced in joint work with Ibai Basabe, Jesús González, and Yuli Rudyak as a generalization of cell complex by allowing non-closed cells. When a topological space X is equipped with a “good” cellular stratification, there is a way to construct a topological category $C(X)$ whose classifying space $BC(X)$ is homotopy equivalent to X . In this talk we review some applications of this fact.

58.

MIMI TSURUGA AND FRANK LUTZ

Constructing Complicated Spheres

TU Berlin, Berlin, Germany

Computational homology packages, such as CHomP or RedHom, avoid large Smith normal form computations by preprocessing input complexes using reduction techniques inspired by Forman’s discrete Morse theory. We present the construction of explicit and complicated triangulations of examples from smooth topology—known as the Akbulut-Kirby spheres—for which reduction is difficult. This infinite series of 4-spheres is based on a handlebody construction via finitely presented groups.

59.

LUIS UGARTE

Strongly Gauduchon metrics and complex deformations

Universidad de Zaragoza - IUMA (Spain)

Strongly Gauduchon metrics were introduced and studied by D. Popovici, and they constitute an important class of metrics lying between the balanced Hermitian metrics and the usual Gauduchon metrics. Associated to any compact complex manifold one can define several cohomological complex invariants as the Aeppli, the Bott-Chern cohomology groups and the terms in the Frölicher spectral sequence. In this talk we focus on the existence of strongly Gauduchon metrics in relation to these complex invariants, as well as the behaviour under holomorphic deformations of the complex structure.

60.

LUCILE VANDEMBROUCQ
On Topological Complexity and related invariants

Universidade do Minho.

I will discuss the relationships between Farber's Topological Complexity and related invariants such as Iwase-Sakai's Monoidal Topological Complexity, Doeraene-El Haouari's relative category and the LS-category of the cofibre of the diagonal map. In particular, I will present some new results obtained in collaboration with José Calcines and José Carrasquel which are related to the Iwase-Sakai conjecture (asserting that Topological Complexity coincides with Monoidal Topological Complexity) and the Doeraene- El Haouari conjecture (asserting that the relative category of a map f coincides with the sectional category when f admits a homotopy retraction).

61.

JOSE ANTONIO VILCHES
**Optimality criteria for discrete Morse functions on 2-complexes
and triangulated 3-manifolds.**

Universidad de Sevilla, Spain

We establish conditions on the existence of optimal (those minimizing the number of critical simplices) discrete Morse functions on 2-complexes and triangulated 3-manifolds. In order to link this study with the homology of the considered complex, we investigate the class of perfect discrete Morse functions, that is, those functions whose numbers of critical simplices are equal to the corresponding Betti number. Also we study the problem of the existence of optimal and non-perfect functions. It is carried out by means of the so called collapse number of the considered complex.

62.

ISMAR VOLIC
**Configuration space integrals and the cohomology of knot and link
spaces**

Wellesley College, Wellesley, U.S.

This talk will survey the use of configuration space integrals in the study of the cohomology of knot and link spaces and in particular their relation to knot invariants of classical knots. More precisely, we will explain the construction of a chain map, given by configuration space integrals, between certain diagram complexes and the deRham complexes of spaces of knots and links in dimension four or more. In dimension three, this map produces all finite type invariants of knots and links. Interesting connections to Milnor invariants of homotopy string links will also be mentioned, as well as the interaction between configuration space integrals and various homotopy-theoretic techniques such as manifold calculus of functors and operads.

63.

ALEXEY VOLOVIKOV
On cohomological index of free G -spaces

Moscow State Technical University of Radio Engineering, Electronics and Automation, Moscow,
Russia

We will consider properties of cohomological index defined on a category of free G -spaces, where G is a finite group. For cyclic groups cohomological index was introduced by Yang and Bourgin via Smith sequences. For general groups it was constructed by Albert Schwarz (homological genus) and Conner & Floyd (cohomological co-index). Conner–Floyd’s index $\text{ind}_L X$ of a free G -space X is defined for any commutative ring with unit L . It can be shown that Schwarz’s homological genus equals $\text{ind}_{\mathbb{Z}} X + 1$, and that $\text{ind}_L X$ satisfies all usual properties of Yang’s homological index (defined for $G = \mathbb{Z}_2$) including the following property:

If $f: X \rightarrow Y$ is an equivariant map of free G -spaces and $\text{ind}_L X = \text{ind}_L Y = n$, then $0 \neq f^*: H^n(Y; L) \rightarrow H^n(X; L)$, in particular f is not homotopic to a constant map.

As a corollary we obtain:

- 1) Cohomological index is stable, i.e. $\text{ind}_L X * G = \text{ind}_L X + 1$, where the join $X * G$ of X and G is considered with diagonal action of G .
- 2) If $f: X \rightarrow X$ is an equivariant selfmap then $0 \neq f^*: H^n(X; L) \rightarrow H^n(X; L)$ where $n = \text{ind}_L X$. In particular f is not homotopic to a constant map.

Second assertion has an overlap with a result of Gottlieb who showed that under some conditions Lefschetz number of f is divisible by the order of G and hence f is not homotopic to zero.

We’ll also consider other properties of index and applications to the space of circumscribed cubes around compact body in Euclidean space.

64.

SINIŠA VREĆICA
On equipartitions of measures.

University of Belgrade, Serbia

Several different versions of equipartition problems appear in Topological combinatorics - equipartitions by hyperplanes, equipartitions in convex sets, equipartitions of necklaces, etc.

In some recent papers by N. Alon et al., M. Lasoń and others it is noticed that splitting necklace theorem is linked in an interesting way with some geometric pattern avoidance problems. Motivated by this, M. Lasoń considered a partition of \mathbb{R}^d in k measurable sets (colors), and asked a question for which k there are two different cubes containing the same measure of every color.

We deal, more generally, with k continuous measures on \mathbb{R}^d and consider the question when there are two (or m) different cubes (or the sets ”of the same type”) in \mathbb{R}^d , which contain the same amount of every measure.

We answer this question, and also provide some examples showing that the obtained answer is the best possible. We notice that our more general setting results in smaller number of measures than conjectured by Lasoń in his case.

65.

ULI WAGNER

Algorithmic and Combinatorial Aspects of Embeddings

IST Austria, Am Campus 1, 3400 Klosterneuburg, Austria

We discuss a number of open questions and results concerning algorithmic and combinatorial aspects of embedding finite simplicial complexes into Euclidean space. Two typical questions are the following: Is there an algorithm that, given as input a finite k -dimensional simplicial complex, decides whether it embeds in d -dimensional space? For $k \geq d/2$, what is the maximum number of k -dimensional faces of a simplicial complex that embeds into d -dimensional space?

Joint work with M. Čadek, M. Krčál, J. Matoušek, F. Sergeraert, E. Sedgwick, M. Tancer, L. Vokřínek

66.

HAN WANG

On the Space of Coverings

University of Illinois, Champaign-Urbana, U.S.A

Consider a compact metric space. We study the structure of the space of r -coverings, that is, the collections of r -balls whose union is the whole space. This problem has not been addressed earlier, despite many applications in engineering. I will present some general results using a Morse-type theory of *max-min* functions and describe the space in simple situations - the space of r -coverings of an interval and of a plaque-formed 2D domain.

67.

FRANK WEILANDT

An algorithm for computing the Conley index of a Poincaré map.

Jagiellonian University, Poland

I will describe a new numerical method to analyse non-autonomous periodic ODE's, which give rise to Poincaré maps. The homological Conley index of a Poincaré map provides essential information on the qualitative behavior of the flow. In particular, it can be applied to prove the existence of periodic orbits or chaotic dynamics. In order to determine the index, one usually numerically follows trajectories of the flow. That causes technical difficulties due to exponentially growing errors of calculations.

In this talk, I will present quite a different approach, based on the construction of an index pair and certain singular chains. It only requires the numerical integration of the flow for a small time, significantly smaller than the period of the ODE. I will present the theoretical and numerical tools, and some promising outputs of our implementation. The theory and ideas for their numerical application are currently developed by Roman Srzednicki and Marian Mrozek.

68.

SHMUEL WEINBERGER
Quantitative problems in pure and applied topology.
University of Chicago, USA.

Many definitions in topology involve existentials over large spaces, and as a result it is hard to put estimates into topological theorems. I will discuss a few examples of the relation between size and topology, and show how such issues arise in both pure topology and in some applications. I expect to emphasize the Lipschitz functional and some questions of Gromov, but I also hope to explain some connections between logic, entropy, and variational problems as well topological sampling problems and their complexities.

69.

MICHAEL WERMAN
Efficient Classification using the Euler Characteristic.
Hebrew University, Israel

We present an object descriptor for supervised classification based on the Euler characteristic of subsets created by thresholding a function defined over the domain at multiple levels. We demonstrate the effectiveness of the descriptor in different domains - images, videos and 3D mesh surfaces. In addition, we propose methods for the efficient calculation of the Euler characteristic for multiple threshold values and calculation of the Euler characteristic in a sliding window.

70.

MATTHEW WRIGHT

Hadwiger Integration and Applications.

Huntington University, Huntington, Indiana, USA; Institute for Mathematics and its Applications,
Minneapolis, Minnesota, USA

The Euler integral makes use of Euler characteristic as a topological invariant, providing a topological notion of *size* for functions. This integration theory has surprising applications to problems arising from sensor networks, as demonstrated by Rob Ghrist and Yuliy Baryshnikov. However, Euler characteristic is only one of $n + 1$ linearly-independent, Euclidean-invariant valuations on “tame” subsets of \mathbb{R}^n . We can integrate real-valued functions with respect to any of these valuations, obtaining what we call *Hadwiger integrals*. These integrals provide various notions of the size of a function, which are useful in areas such as sensor networks, image processing, and cellular dynamics. Furthermore, Hadwiger integrals can be defined on random fields, which may help deal with uncertainty in applied situations. This talk will explain the theory of Hadwiger integration and discuss some of its challenges, applications, and opportunities for future work.

71.

KRZYSZTOF ZIEMIAŃSKI

Path spaces on skeleta of tori.

University of Warsaw, Poland

A path on a torus $T^n := \mathbb{R}^n/\mathbb{Z}^n$ is directed, if and only if it lifts to a path on \mathbb{R}^n which is non-decreasing on all coordinates. The main goal of my talk is to present the calculation of the homotopy type of the space of directed paths on skeleta of the torus T^n . The calculation uses iterated homotopy decomposition techniques which seem to be useful when solving similar problems. This result provides a description of a behavior of a computer program containing n processes which periodically acquire and release a single semaphore of a given arity.

This is a common work with Martin Raussen.

72.

RADE T. ŽIVALJEVIĆ

Computational topology and effective obstruction theory

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The problem of calculating topological obstructions to the existence of equivariant maps is of great relevance for computational topology. The focus is naturally on those features of the problem where topology and computational mathematics interact in an essential way (algorithmic aspects of the problem, explicit procedures suitable for large scale calculations).

We propose an approach to “computational primary obstruction theory” based on G -manifold complexes with the following characteristic features.

- (1) One works with manifold G -complexes which are more general and often more economical than G -CW-complexes.
- (2) Given a G -space (manifold) X , the associated G -manifold complex arises through the iteration of an effective fundamental domain - geometric boundary’ procedure.
- (3) The generators of G -modules are fundamental classes and the boundaries and chain maps are evaluated as mapping degrees.

As an application we compute a primary cohomological obstruction to the existence of an equipartition for j mass distributions in \mathbb{R}^d by two hyperplanes in the case $2d - 3j = 1$. The central new result is that such an equipartition always exists if $d = 6 \cdot 2^k + 2$ and $j = 4 \cdot 2^k + 1$ which for $k = 0$ reduces to the main result of the paper P. Mani-Levitska et al., *Topology and combinatorics of partitions of masses by hyperplanes*, *Adv. Math.* 207 (2006), 266–296. The theorem follows from a Borsuk-Ulam type result claiming the non-existence of a \mathbb{D}_8 -equivariant map $f : S^d \times S^d \rightarrow S(W^{\oplus j})$ for an associated real \mathbb{D}_8 -module W . This is an example of a genuine combinatorial geometric result which involves $\mathbb{Z}/4$ -torsion in an essential way and cannot be obtained by the application of more standard methods (Stiefel-Whitney classes).

- [1] R.T. Živaljević. Topological methods. Chapter 14 in *Handbook of Discrete and Computational Geometry*, J.E. Goodman, J. O’Rourke, eds, Chapman & Hall/CRC 2004, 305–330.
- [2] R.T. Živaljević. Equipartitions of measures by two hyperplanes, arXiv:1111.1608 [math.CO].

