Summability of formal solutions of 1st order nonlinear ODE related to linearization problem

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Let us consider to linearize 1st order nonlinear ODE. Then the changes $x+u(x)$ satisfy the following equation:

(E)
$$x\frac{d}{dx}u(x) - u(x) = R(x + u(x))$$

where a function $R(X)$ is a holomorphic function near $X = 0$ and $R(X) = O(X^2)$.

In this talk we consider

(Ex)
$$\eta^{-1}x\frac{\partial}{\partial x}u(x,\eta) - u(x,\eta) = (x + u(x,\eta))^2.$$

For the example (Ex) we have a formal solution $\widetilde{u}(x,\eta) = \sum_{k=0}^{\infty} u(x)\eta^{-k}$ with $u_k(x) \in \mathcal{O}(D_R)$ and study Borel summability of the solution $\widetilde{u}(x,\eta)$.