

ANALYSIS ON THE WIENER-POISSON SPACE AND ITS APPLICATION TO THE ASYMPTOTIC EXPANSION

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Let F be a Wiener-Poisson functional. In case F satisfies the (ND) condition, it is shown that the characteristic function of F satisfies

$$E[e^{i\xi \cdot F}] = O((1 + |\xi|^2)^{-q_0 n/2})$$

as $|\xi| \rightarrow \infty$ for any n . Here $0 < q_0 < 1$. This implies that F has a C^∞ -density function $p_F(y)$, since

$$\partial_x^l p_F(x) = \left(\frac{1}{2\pi}\right)^d \int e^{-ix \cdot \xi} (-i\xi)^l E[e^{i\xi \cdot F}] d\xi, l = 0, 1, 2, \dots$$

In case $F = X(t)$, where $X(t)$ denotes a jump-diffusion process such that $X(0) = x$, the density function is written by $p_t(x, y)$. In this way, the density function as an analytic object (fundamental solution of some integro-differential equation) can be obtained by using Malliavin calculus of jump(-diffusion) type.

The density function of F at $x \in \mathbf{R}^d$ can formally be defined by $E[\delta_x(F)] = \langle \delta_x(F), 1 \rangle$, where Dirac's delta function δ_x is an element of the tempered distributions \mathcal{S}' . In case $F = X(t)$ where $X(t)$ is as above, the density function $p_t(x, y)$ of $X(t)$ is justified to be represented by $p_t(x, y) = E[\delta_y(X(t))]$.

We apply these results to lead asymptotic expansion of a model process appearing in the H-H (Hodgkin-Huxley) model.

The background theory of the former half of the content of this talk is written in the book [2]. For the latter half, it will be written in [4].

REFERENCES

- [1] M. Hayashi and Y. Ishikawa, Composition with distributions of Wiener-Poisson variables and its asymptotic expansion, *Math. Nach.* 285, No. 5-6, 619-658 (2012).
- [2] Y. Ishikawa, *Stochastic calculus of variations for jump processes*, Studies in Mathematics 54, Walter-de-Gruyter, Berlin, 2013.
- [3] Y. Ishikawa and H. Kunita, Malliavin calculus on the Wiener-Poisson space and its application to canonical SDE with jumps, *Stochastic processes and their applications* 116 (2006) 1743–1769.
- [4] Y. Ishikawa, Nerve cell model and asymptotic expansion, preprint, 2013.
- [5] H. Kunita, Analysis of nondegenerate Wiener-Poisson functionals and its application to Itô's SDE with jumps, *Sankhya* 73 (2011), 1–45.
- [6] N. Yoshida, Conditional expansions and their applications, *Stochastic processes and their applications* 107 (2003) 53–81.