

A SYMBOLIC APPROACH TO MULTIVARIATE POLYNOMIAL LÉVY PROCESSES

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A class of multivariate processes, called *polynomial processes*, has been recently introduced in [1] as follows. Assume S a closed subset of \mathbb{R}^d the state space of a stochastic process \mathbf{X}_t . Then there exists a multivariate polynomial $Q(\mathbf{x}, t)$ in

$$\text{Pol}_{\leq m}(S) = \left\{ \sum_{|\mathbf{k}|=0}^m c_{\mathbf{k}} \mathbf{x}^{\mathbf{k}} \mid \mathbf{x} \in S, c_{\mathbf{k}} \in \mathbb{R} \right\}, \quad (1)$$

where $\mathbf{x}^{\mathbf{k}} = x_1^{k_1} x_2^{k_2} \cdots x_d^{k_d}$ and $c_{\mathbf{k}} = c_{k_1, k_2, \dots, k_d}$, such that $E[Q(\mathbf{X}_t, t) | \mathbf{X}_s] = Q(\mathbf{X}_s, s)$ for $s \leq t$. The martingales $\{Q(\mathbf{X}_t, t)\}$ are called *polynomial processes*. These processes are employed for the pricing and the hedging of some bounded measurable European claims together with the reduction–variance method. In [1], the attention is essentially focused on the properties shared by the class of stochastic processes $Q(\mathbf{X}_t, t)$, as for example affine processes or Feller processes with quadratic squared diffusion coefficients. In order to characterize $Q(\mathbf{x}, t)$, Haar measure and zonal polynomials are involved, hardly to be managed by a computational point of view. So the computation of their coefficients usually involves matrix exponentials. In the univariate case, the polynomials (1) have been deeply analyzed by different authors, see [4] and references therein. They are called *time-space harmonic* polynomials. For Lévy processes \mathbf{X}_t , the main advantage of employing the polynomial process $Q(\mathbf{X}_t, t)$ is the martingale property, fundamental in the martingale pricing [3], which not necessarily holds for Lévy processes.

This contribution focuses on a symbolic representation of multivariate Lévy processes, relied on an algebraic setting known in the literature as the classical umbral calculus [2]. Particular emphasis is placed on the role played by cumulants, this giving rise to a plain symbolic representation which reduces to few fundamental statements the main results on multivariate Lévy processes. For example, a closed formula for $Q_{\mathbf{v}}(\mathbf{x}, t)$, with $|\mathbf{v}| = m$, is given in the following theorem and its implementation in any symbolic software is particularly suited.

Theorem 1. *For all $\mathbf{v} \in \mathbb{N}_0^d$, the family of polynomials*

$$Q_{\mathbf{v}}(\mathbf{x}, t) = E[(\mathbf{x} - t \cdot \boldsymbol{\mu})^{\mathbf{v}}] \in \mathbb{R}[x_1, \dots, x_d] \quad (2)$$

is time-space harmonic with respect to $\{t \cdot \boldsymbol{\mu}\}_{t \geq 0}$, symbolic representation of the Lévy process \mathbf{X}_t .

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