ONE DIMENSIONAL COMPLETELY ASYMMETRIC MARKOV PROCESSES

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In the early 50's, Feller [4] characterized fully the analytical structure of one dimensional diffusion processes. In particular, he constructed their Green functions in terms of the two positive fundamental solutions of a second order differential equations associated to their infinitesimal generators. Following Darling and Siegert [2], he also characterized completely the Laplace transform of their first exit times from an interval in terms of these two fundamental solutions. These type of characterization has been also achieved for the class of spectrally negative Lévy processes and some related processes. There is a substantial literature devoted to these problems in this framework, we mention the works of Takacs [7], Suprun [6], Bertoin [1], Kyprianou and Palmowski [5] and Doney [3] to name but a few. In this work, we aim to pursue Feller's program for the class of one dimensional completely asymmetric Markov processes, that is strong Markov processes having jumps only in one direction. We will present an original methodology based mostly on potential theoretical arguments to characterize the Laplace of their first exit times from an interval which may occur by a jump. We will also describe, in terms of fundamental solutions, their resolvent densities whose existence of a nice version is provided. We illustrate our approach by recovering easily the well-known fluctuations identities for spectrally negative Lévy processes.

References

[1] J. Bertoin. Exponential decay and ergodicity of completely asymmetric Lévy processes in a finite interval. *Ann. Appl. Probab.*, 7(1):156–169, 1997.

[2] D.A. Darling and A.J.F. Siegert. The first passage problem for a continuous Markov process. *Ann. Math. Stat.*, 24:624-639, 1953.

[3] R.A. Doney. Fluctuation theory for Lévy processes, *Lecture Notes in Mathematics*, Springer, Berlin, 2007.

[4] W. Feller. Diffusion processes in one dimension. *Trans. Amer. Math. Soc.*, 77:1–31, 1954.

[5] A.E. Kyprianou and Z. Palmowski. A martingale review of some fluctuation theory for spectrally negative Lévy processes. In *Séminaire de Probabilités XXXVIII*, volume 1857 of *Lecture Notes in Math.*, pages 16–29. Springer, Berlin, 2005.

[6] V.N. Suprun. The ruin problem and the resolvent of a killed independent increment process. *Ukrain. Mat. Ž.*, 28(1):53–61, 1976.

[7] L. Takács. *Combinatorial Methods in the Theory of Stochastic Processes.* Wiley, New York, 1966.