MARTINGALE SOLUTIONS OF THE STOCHASTIC HYDRODYNAMIC-TYPE EQUATIONS DRIVEN BY LÉVY NOISE

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Let $\mathcal{O} \subset \mathbb{R}^d$, $d = 2, 3$, be an open connected possibly unbounded subset with smooth boundary $\partial\mathcal{O}$. Let $\mathbb{H} \subset L^2(\mathcal{O}; \tilde{d})$ and $\mathbb{V} \subset H^1(\mathcal{O}; \tilde{d})$, where $\tilde{d} \in \mathbb{N}$, be two Hilbert spaces such that $\mathbb{V} \subset \mathbb{H}$, the embedding being continuous. We consider the following stochastic equation

$$
\begin{aligned}
u(t) &+ \int_0^t \big[\mathcal{A}u(s) + \mathcal{B}(u(s)) + \mathcal{R}u(s)\big]\, ds = u_0 + \int_0^t f(s)\, ds \\
&+ \int_0^t \int_{Y_0} F(s, u(s^-); y)\tilde{\eta}(ds, dy) + \int_0^t \int_{Y\setminus Y_0} F(s, u(s^-); y)\eta(ds, dy) \\
&+ \int_0^t G(s, u(s))\, dW(s), \quad t \in (0, T).
\end{aligned}
$$

In this equation $\mathcal{A}, \mathcal{B}, \mathcal{R}$ are maps defined in the spaces $\mathbb{H}$ or $\mathbb{V}$, satisfying appropriate conditions. Moreover, W stands for a cylindrical Wiener process on a separable Hilbert space and η is a time-homogeneous Poisson random measure on a measurable space $(Y, \mathcal{Y})$ with a σ-finite intensity measure ν and $\nu(Y \setminus Y_0) < \infty$. We prove the existence of a martingale solution. The proof is based on the Faedo-Galerkin approximation. We use appropriate tightness criterion and a certain version of the Skorokhod Theorem in non-metric spaces. The abstract framework is applied to the stochastic Navier-Stokes equations, magneto-hydrodynamic equations (MHD) and Boussinesq equations.

References

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