ON THE SELF-DECOMPOSABILITY OF THE FRÉCHET DISTRIBUTION

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Let $\{\Gamma_t,\, t \geq 0\}$ be the Gamma process. Using a moment identification due to Bertoin and Yor [4] we observe that for every $t > 0$ and $\alpha \in (0,1)$ the random variable $\Gamma_t^{-\alpha}$ is distributed as the exponential functional of some spectrally negative Lévy process. This entails that all size-biased samplings of Fréchet distributions are self-decomposable and that the extreme value distribution F_ξ is infinitely divisible if and only if $\xi \notin (0,1)$, solving problems raised by Steutel [19] and Bondesson [6]. We also review different analytical and probabilistic interpretations of the infinite divisibility of $\Gamma_t^{-\alpha}$ for $t, \alpha > 0$.

Theorem 1. *For every $\alpha \in (0,1)$ and $t > 0$, the random variable $\Gamma_t^{-\alpha}$ is self-decomposable.*

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