ASYMPTOTIC INDEPENDENCE OF THREE STATISTICS OF THE MAXIMAL INCREMENTS OF RANDOM WALKS AND LÉVY PROCESSES

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Let $H(x) = \inf\{n : \exists\, k < n : S_n - S_k > x\}$

$$R_n = \max_{m \in \{0,\dots,n\}} \{S_n - S_m\}, \qquad R_n^* = \max_{m,k \in \{0,\dots,n\}, m \le k} \{S_k - S_m\}$$

and $O_x = R_{H(x)} - x$. In this talk we show that under Cramér's condition on the step-size distribution of S, the statistics R_n, $R_n^* - y$ and O_{x+y} are asymptotically independent as $\min\{n, y, x\} \uparrow \infty$. Furthermore, we establish a novel Spitzer-type identity characterising the limit law O_∞ in terms of the one-dimensional marginals of S. If $y = \gamma^{-1} \log n$, where γ is the Cramér coefficient, our results together with the classical theorem of Iglehart [1] imply the existence of a joint weak limit of the three statistics and identify its law. As corollary we obtain a new factorization of the exponential distribution. We prove analogous results for the corresponding statistics of a Lévy process.

References

[1] D.L. Iglehart. Extreme values in the GI/G/1 queue. *Ann. Math. Statist.*, 43:627-635, 1972.
[2] A. Mijatovic, M. Pistorius. Asymptotic independence of three statistics of the maximal increments of random walks and Lévy processes *Preprint*, 2012.