FUNCTIONAL LIMIT THEOREM FOR PARTIAL MAXIMA FOR STATIONARY SYMMETRIC α-STABLE PROCESSES GENERATED BY CONSERVATIVE FLOWS

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We derive the functional limit theorem for partial maxima $\max_{1\le k\le \lfloor nt\rfloor} |X_k|$, $n = 1, 2, \dots$, $t \ge 0$ of stationary α-stable processes of a certain integral representation. First of all, we assume that the process is generated by a conservative flow. This assumption relates to the long memory in the process. In particular, the length of memory observed in the process is significantly longer than that in the process generated by a dissipative flow (e.g. moving averages with regularly varying innovations). Moreover, the assumption that the flow is pointwise dual ergodic allows us to quantify the memory length in the process by a parameter $\beta \in (1/2, 1)$. Consequently, the normalizing constants (b_n) of the partial maxima forms a regularly varying sequence with index β/α. Furthermore, the limiting process of the normalized partial maxima is no longer a classical extremal process and is, in fact, a new class of self-similar α-Fréchet process with max-stationary increments. The functional limit theorem is established in the space $D[0,\infty)$ equipped with the Skorohod M_1-topology; however, if the integral representation of the process takes a simple form, we can strengthen the topology of $D[0,\infty)$ to the Skorohod J_1-topology. This is because, in that case, $b_n^{-1} \max_{1\le k\le \lfloor nt\rfloor} |X_k|$ possesses "single jump structure," while in a general case, it has "multiple jump structure."

Theorem 1. *Let T be a conservative, ergodic, measure preserving, and pointwise dual ergodic map on a σ-finite infinite measure space $(E, \mathcal{E}, \mu)$. Let M be a symmetric α-stable random measure, $0 < \alpha < 2$, on $(E, \mathcal{E})$ with control measure μ. Let $f : E \to \mathbb{R}$ be a $L^\alpha(\mu)$-integrable function that is supported by some measurable set A of finite μ-measure.*

We consider a class of stationary symmetric α-stable process

$$X_n = \int_E f \circ T^n(x) dM(x), \quad n = 1, 2, \dots$$

Then, $\mathbf{X} = (X_1, X_2, \dots)$ satisfies

$$b_n^{-1} \max_{1\le k\le \lfloor nt\rfloor} |X_k| \Rightarrow Z_{\alpha,\beta}(t) \quad \text{in } D[0,\infty)\,,$$

where (b_n) is a regularly varying normalizing sequence with index β/α for some $\beta \in (1/2, 1)$, and $Z_{\alpha,\beta}(t)$ is an α-Fréchet process with control measure $\nu_\beta(dx) = \beta x^{\beta-1}dx$, $x > 0$.
Here, $\Rightarrow$ means weak convergence in the space $D[0,\infty)$, equipped with the Skorohod M_1-topology.

Moreover, if $f = \mathbf{1}_A$, then the above convergence occurs in the Skorohod J_1-topology.