THE DISTRIBUTION OF THE ROSENBLATT PROCESS – EXAMPLES OF DISTRIBUTIONS IN THE THORIN CLASS

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Let $\{B(s), s \in \mathbb{R}\}$ be the standard Brownian motion. The Rosenblatt and the non-symmetric Rosenblatt processes are defined, respectively, as, for $t \geq 0$,

$$Z_D(t) = C(D) \int_{\mathbb{R}^2}' \left(\int_0^t (u - s_1)_+^{-(1+D)/2} (u - s_2)_+^{-(1+D)/2} du \right) dB(s_1) dB(s_2)$$

and

$$Z_{D_1,D_2}(t) = C(D_1, D_2) \times$$
$$\times \int_{\mathbb{R}^2}' \left(\int_0^t (u - s_1)_+^{-(1+D_1)/2} (u - s_s)_+^{-(1+D_2)/2} du \right) dB(s_1) dB(s_2),$$

where $0 < D, D_1 \neq D_2 < \frac{1}{2}$, $\int_{\mathbb{R}^2}'$ is the integral over $\mathbb{R}^2$ except the hyperplane $s_1 = s_2$, and $C(D)$ and $C(D_1, D_2)$ are the respective normalizing constants.

As a class of infinitely divisible distributions on $\mathbb{R}^d$, the Thorin class $T(\mathbb{R}^d)$ is defined as follows. Call Γx an elementary gamma random variable in $\mathbb{R}^d$ if $x (\neq 0) \in \mathbb{R}^d$ and Γ is a gamma random variable. Then $T(\mathbb{R}^d)$ is defined as the smallest class of distributions on $\mathbb{R}^d$ that contains all elementary gamma distributions on $\mathbb{R}^d$ and is closed under convolution and weak convergence. When $d = 1$, $\mu \in T(\mathbb{R}_+)$ is called generalized gamma convolution (GGC) and $\mu \in T(\mathbb{R})$ is called extended generalized gamma convolution (EGGC).

In this talk, we show that any finite dimensional distribution of the process $\{Z_{D_1,D_2}(t)\}$ belongs to the Thorin class.

As byproducts, we can see that when $d = 1$ any marginal distribution of $Z_{D_1,D_2}(t)$ is unimodal (since it is selfdecomposable) and when $d \geq 1$ it is represented by a single integral with respect to some Lévy process, although it has the representation by a double integral with respect to Brownian motion.

We also mention many other examples of GGCs and EGGCs in several different problems, which show how rich the class $T(\mathbb{R})$ is. Among others, there are the length of random excursion of some Bessel processes by Bertoin-Fujita-Roynette-Yor (2006), the stationary solution of some continuous state branching processes with immigration) by Handa (2012) and the inverse local time at zero of some diffusion processes by Takemura-Tomisaki (2012) (see also Chapter 14 of Schilling-Song-Vondraček (2010)).