USEFUL MARTINGALES FOR STOCHASTIC STORAGE PROCESSES WITH LÉVY-TYPE AND MARKOV ADDITIVE INPUT

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This talk will summarize certain exponential martingale tools that are useful in the study of Lévy storage processes and in particular reflected Lévy and Markov additive processes and such processes having secondary jump inputs. A statement of the results will be given followed by various examples of their applicability (as much as time permits). Basically this will be a summary of [2, 3, 4, 5], where the starting point is the martingale

$$M_t = \psi(\alpha)\int_0^t e^{i\alpha Z_s}ds + e^{i\alpha Z_0} - e^{i\alpha Z_t} + i\alpha\int_0^t e^{i\alpha Z_s}dY_s^c + \sum_{0<s\le t} e^{i\alpha Z_s}\left(1 - e^{-i\alpha\Delta Y_s}\right)$$

and its Laplace-Stieltjes counterpart when there are no negative jumps, where X is a (càdlàg) Lévy process, Y is (càdlàg) adapted and VF, $Z = X + Y$, $\Delta Y_s = Y_s - Y_{s-}$ and $Y_t^c = Y_t - \sum_{0<s\le t}\Delta Y_s$.

Recently it has been observed in [2] that this is always an L^2 martingale without any further conditions (such as the ones initially described in [5]) and furthermore it satisfies $M(t)/t \to 0$ as $t \to \infty$ both in L^2 and almost surely. Moreover, it keeps having these properties if X is replaced by $\sum_{i=1}^K \int_{[0,t]} I_{s-}^i dX_s^i$ where I^i are (càdlàg) adapted and bounded and $(X^1, \ldots, X^K)$ is some multivariate Lévy process. This turns out to be useful for studying certain (non-Markovian) modulated reflected Lévy processes with or without secondary jump inputs as discussed in

References

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