APPROXIMATION OF MARKOV SEMIGROUPS VIA FEYNMAN FORMULAE

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We present a new approach to approximate Markov evolution. This approach is based on representations of evolution semigroups by *Feynman formulae*, i.e. by limits of iterated n-fold integrals when n tends to infinity. One succeeds to obtain Feynman formulae for different types of evolution on various geometrical structures. Sometimes it is possible to get Feynman formulae containing only integrals of elementary functions. Such Feynman formulae can be used for direct calculations; they allow to approximate transition probabilities of underlying stochastic processes and hence to simulate these processes, to calculate solutions of related evolution equations numerically, to model the corresponding dynamics.

The limits in Feynman formulae often coincide with functional integrals with respect to probability measures or Feynman type pseudomeasures. Representations of evolution semigroups by functional integrals with respect to probability measures are usually called *Feynman–Kac formulae*. Functional integrals with respect to Feynman type pseudomeasures are usually called *Feynman path integrals* and they are an important tool of quantum physics. Feynman formulae provide a new way to establish Feynman-Kac formulae, to calculate functional integrals, to find new connections between quantum mechanics and stochastic analysis.

We demonstrate the approach of Feynman formulae for approximation of multiplicative and additive perturbations of Markov evolution, for approximation of Markov evolution in a domain with Dirichlet boundary conditions, for approximation of some Feller semigroups. We present also some Feynman–Kac formulae and Feynman path integrals related to the considered Feynman formulae. We consider evolution on different geometric structures, e.g., domains in Euclidean spaces and in Riemannian manifolds.

Some of the considered results are obtained in collaboration with O.G. Smolyanov, R.L. Schilling and M. Grothaus. The investigations have been supported by the Grant of the President of Russian Federation MK-4255.2012.1, the Grant of the Ministry of education and science of Russian Federation 14.B37.21.0370, DFG, DAAD and Erasmus Mundus.

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