

ON EXTREMA OF STABLE PROCESSES

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Real analysis offers a multitude of examples of functions with “strange” analytic properties, such as a continuous function on $[0, 2\pi]$, for which the corresponding Fourier series diverges almost everywhere. All of these examples are somewhat artificial, in the sense that they have to be specifically constructed in order to satisfy given property, and in many cases the proof of existence of such functions is non-constructive at all. Overall, it would seem unlikely that such functions could arise in “real-life” applications. This partly explains why the supremum of a stable process is such a fascinating object: On the one hand it arises naturally from many useful applications of stable processes in Mathematics, Physics, Chemistry and other Sciences, and on the other hand its distribution has the most unusual analytic properties.

We will review the origin and development of the problem of finding the distribution of extrema of stable processes [2, 3, 4, 7], concentrating on connections with Number Theory. Then we will discuss some recent results [1, 5, 6, 8, 9, 10] on the Wiener-Hopf factorization of stable processes and on the series representation of the density of the supremum. We will also present several open problems and discuss possible directions for future research, and we will emphasize connections with other fields of Mathematics.

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