## STRONG LIMIT THEOREMS FOR SELF-SIMILAR RANDOM FIELDS

VITALII MAKOGIN

We consider self-similar random fields and investigate their sample path properties for upper and lower functions in cases of non-decreasing and non-increasing functions. We present some examples of iterated log-type limits for Gaussian self-similar random fields and fractional Brownian sheets. Random field $\{X(t), t = (t_1, \ldots, t_n) \in \mathbb{R}^n\}$ is self-similar with index $\mathbf{H} = (H_1, \ldots, H_n) \in \mathbb{R}^n_+$ if for all $a_1 > 0, \ldots, a_n > 0$, the finite-dimensional distributions of

$$a_1^{-H_1} \cdots a_n^{-H_n} X(a_1 t_1, \ldots, a_n t_n), (t_1, \ldots, t_n) \in \mathbb{R}^n$$

are identical to the finite-dimensional distributions of $X(t_1, \ldots, t_n)$. The aim is to prove strong limit theorems for sample paths of self-similar random fields. In [3] Kono considers such theorems for self-similar random processes. We generalize his results.

Let $\left\{X\left(t_1, t_2\right);\ t = (t_1, t_2) \in \mathbb{R}^2_+\right\}$ be a real valued separable, measurable, stochastically continuous self-similar random field of order $(H_1, H_2), H_1 > 0, H_2 > 0$. Set

$$Y\left(\omega\right) = \sup_{\substack{0 \le t_1 \le 1,\\ 0 \le t_2 \le 1}} \left|X\left(t_1, t_2, \omega\right)\right|.$$

**Theorem 1.** *Let $f = f(x)$ be a positive, continuous, non-decreasing function defined on $\mathbb{R}_+$. Suppose $\mathbf{E}\left[f\left(Y\right)\right]$ is finite, $\varphi\left(x_1, x_2\right)$ is a positive continuous function defined on $\mathbb{R}^2_+$ and satisfying the following conditions:*

(1) *$\varphi\left(x_1, x_2\right)$ is non-decreasing in each variable,*

(2) $\lim_{x \downarrow 1} \sup_{n,m=1,2,\ldots} \frac{\varphi(x^n, x^m)}{\varphi(x^{n-1}, x^{m-1})} = c < +\infty$,

(3) $\int_1^{+\infty} \frac{dx}{x f(\varphi(x,x))} < +\infty$;

*then*

$$\overline{\lim_{\substack{s_1 \to +\infty\\ s_2 \to +\infty}}} \frac{\left|X\left(s_1, s_2\right)\right|}{s_1^{H_1} s_2^{H_2} \varphi\left(s_1, s_2\right)} \le c \qquad a.s.$$

### References

[1] Embrechts, Paul and Maejima, Makoto *Selfsimilar Processes.* Princeton University Press, 2001.

[2] Marc G. Genton, Olivier Perrinc & Murad S. Taqqu. *Self-Similarity and Lamperti Transformation for Random Fields.* volume 23(3) of *Stochastic Models.* Taylor & Francis 2007, p.397-411.

[3] N. Kôno *Iterated log type strong limit theorems for self-similar precesses*, volume 59 of *Proceedings of the Japan Academy, Series A*, 1983, p. 85-87.

[4] D. M. Mason and Z. Shi. *Small deviations for some multi-parameter Gaussian processes.* volume 14.1 of *Journal of Theoretical Probability.* 2001, 213-239.