STRONG LIMIT THEOREMS FOR SELF-SIMILAR RANDOM FIELDS

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We consider self-similar random fields and investigate their sample path properties for upper and lower functions in cases of non-decreasing and non-increasing functions. We present some examples of iterated log-type limits for Gaussian self-similar random fields and fractional Brownian sheets. Random field \( \{X(t), t = (t_1, \ldots, t_n) \in \mathbb{R}^n\} \) is self-similar with index \( \mathbf{H} = (H_1, \ldots, H_n) \in \mathbb{R}_+^n \) if for all \( a_1 > 0, \ldots, a_n > 0 \), the finite-dimensional distributions of

\[
a_1^{-H_1} \cdots a_n^{-H_n} X(a_1 t_1, \ldots, a_n t_n), (t_1, \ldots, t_n) \in \mathbb{R}^n
\]

are identical to the finite-dimensional distributions of \( X(t_1, \ldots, t_n) \). The aim is to prove strong limit theorems for sample paths of self-similar random fields. In [3] Kono considers such theorems for self-similar random processes. We generalize his results.

Let \( \{X(t_1, t_2); t = (t_1, t_2) \in \mathbb{R}_+^2\} \) be a real valued separable, measurable, stochastically continuous self-similar random field of order \( (H_1, H_2) \), \( H_1 > 0, H_2 > 0 \). Set

\[
Y(\omega) = \sup_{0 \leq t_1 \leq 1, 0 \leq t_2 \leq 1} |X(t_1, t_2, \omega)|.
\]

**Theorem 1.** Let \( f = f(x) \) be a positive, continuous, non-decreasing function defined on \( \mathbb{R}_+ \). Suppose \( E[f(Y)] \) is finite, \( \varphi(x_1, x_2) \) is a positive continuous function defined on \( \mathbb{R}_+^2 \) and satisfying the following conditions:

1. \( \varphi(x_1, x_2) \) is non-decreasing in each variable,
2. \( \lim_{x \to 1} \sup_{n, m = 1, 2, \ldots} \frac{\varphi(x^n, x^m)}{\varphi(x^{n-1}, x^{m-1})} = c < +\infty \),
3. \( \int_1^{+\infty} \frac{dx}{xf(\varphi(x,x))} < +\infty \); then

\[
\lim_{s_1 \to +\infty} \lim_{s_2 \to +\infty} \frac{|X(s_1, s_2)|}{s_1^{-H_1} s_2^{-H_2} \varphi(s_1, s_2)} \leq c \text{ a.s.}
\]

**REFERENCES**