LONG-RANGE DEPENDENCE MEETS SHORT-RANGE DEPENDENCE: MULTIVARIATE LIMIT THEOREMS

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It is well known that the normalized partial sum of a short-range dependent (SRD, or say weakly dependent, e.g., independent) stationary sequence converges weakly to Brownian motion. A long-range dependent (LRD) stationary sequence, however, typically results in limits other than Brownian motion, and when the sequence is nonlinear, the limit may not even be Gaussian. An example of this kind is the *multilinear polynomial-form process* (or *discrete-chaos process*) defined as:

$$X(n) = \sum_{i_1,\ldots,i_k}{}' a_{i_1} \ldots a_{i_k} \epsilon_{n-i_1} \ldots \epsilon_{n-i_k},$$

where the prime $'$ indicates that we don't sum on diagonals: $i_p = i_q, p \neq q$, $\{\epsilon_i\}$ are i.i.d. noise, and $\{a_i\}$ is regularly varying. Depending on the decaying rate of $\{a_i\}$ and the order k, $X(n)$ may be SRD or LRD. When $X(n)$ is LRD, the limit process is the so-called *Hermite process* of order k, where if $k = 1$ the Hermite process is fractional Brownian motion, and if $k \geq 2$, the law of the process belongs to the kth-order Wiener chaos and is thus non-Gaussian.

We study the multivariate limit for a vector $\mathbf{Y}_N$ made up of normalized partial sums of such $X(n)$'s defined through the same $\{\epsilon_i\}$ but different $\{a_i\}$'s. The most interesting case is when the vector is a mixture of SRD and LRD components. An asymptotic independence feature is observed in the mixed limit. The main result is the following:

Theorem 1. Break $\mathbf{Y}_N$ into 3 parts:

$$\mathbf{Y}_N = (\mathbf{Y}_{N,S_1}, \mathbf{Y}_{N,S_2}, \mathbf{Y}_{N,L}),$$

$\mathbf{Y}_{N,S_1}$: $J_{S_1}-$dimensional, every component is SRD, order $k_{j,S_1} = 1$,
$\mathbf{Y}_{N,S_2}$: $J_{S_2}-$dimensional, every component is SRD, order $k_{j,S_2} \geq 2$,
$\mathbf{Y}_{N,L}$: J_L-dimensional, every component is LRD.

Then (" $\xrightarrow{f.d.d.}$ " denotes convergence in finite-dimensional distributions)

$$\mathbf{Y}_N(t) = (\mathbf{Y}_{N,S_1}(t), \mathbf{Y}_{N,S_2}(t), \mathbf{Y}_{N,L}(t)) \xrightarrow{f.d.d.} (\mathbf{W}(t), \mathbf{B}(t), \mathbf{Z}^{\mathbf{k}_L}_{\mathbf{d}_L}(t)),$$

$\mathbf{B}(t)$: a multivariate Brownian motion,
$\mathbf{Z}^{\mathbf{k}_L}_{\mathbf{d}_L}(t)$: a multivariate Hermite process,
$\mathbf{W}(t) := (W(t), \ldots, W(t))$, where $W(t)$ is the random integrator defining $\mathbf{Z}^{\mathbf{k}_L}_{\mathbf{d}_L}(t)$.

Moreover, $\mathbf{B}(t)$ is independent of $(\mathbf{W}(t), \mathbf{Z}^{\mathbf{k}_L}_{\mathbf{d}_L}(t))$.

References

[Bai and Taqqu(2012)] S. Bai and M.S. Taqqu. Multivariate limit theorems in the context of long-range dependence. *arXiv preprint arXiv:1211.0576v2*, 2012.

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