AVERAGING FOR STOCHASTIC REACTION DIFFUSION EQUATIONS WITH COEFFICIENTS OF POLYNOMIAL GROWTH

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Let H denote a separable Hilbert space, X a separable Banach space and define

$$M(H,X) := \{L : H \to X : \ L \text{ is linear bounded and } \gamma\text{–radonifying } \}.$$

Let $w(t)$, $t \geq 0$ be an H-cylindrical Wiener process on a probability space. Assume that for each $\alpha \in I$, A_α is the infinitesimal generator of an analytic semigroup in X and there exists a Banach space B such that

$$D\left(A_\alpha^\delta\right) \hookrightarrow B \hookrightarrow X,$$

where the first imbedding is uniform with respect to $\alpha \in I$.

Sufficient conditions are imposed on the maps A_α, $F_\alpha : [0,\infty) \times B \to B$, $A_\alpha^{-\sigma} G_\alpha : [0,\infty) \times B \to M(H,X)$ and positive constants δ, σ such that weak martingale solutions to

$$\begin{cases} \mathrm{d}u_\alpha + A_\alpha u_\alpha \mathrm{d}t = F_\alpha(t,u_\alpha)\mathrm{d}t + G_\alpha(t,u_\alpha)\mathrm{d}w(t) \\ \qquad\qquad u_\alpha(0) = \varphi_\alpha \end{cases} \tag{1}$$

exist and the following two theorems hold:

Theorem 1. *The family* $\{u_\alpha\}_{\alpha\in I}$ *of solutions to equation* (1) *is tight on* $C([\tau,T];B)$ *for* $\tau \in (0,T)$.

Theorem 2. *The solution to the limit problem of equation* (1) *exists, and as* $\alpha \to 0$, u_α *converges weakly to the solution.*

Remarks.

- For $L \in M(H,X)$, let ν_L denote the Guassian measure defined on the Borel σ-algebra $B(X)$ generated by L . Then $M(H,X)$ is a normed vector space with norm
$$|L|_{M(H,X)} := \left\{ \int_X |x|^2 \,\mathrm{d}\nu_L(x) \right\}^{\frac{1}{2}}, \quad L \in M(H,X).$$
Moreover, $M(H,X)$ is a separable Banach space, see [1];

- Existence results to equation (1) are established in [2];

- For the case X is a Hilbert space consult [3] and the sequel [4].

References

[1] A.L. Neidhardt. *Stochastic integrals in 2-uniformly smooth Banach spaces*, University of Wisconsin. 1978

[2] Z. Brzeźniak and D. Gątarek. *Martingale solutions and invariant measures for stochastic evolution equations in Banach spaces*, volume 84 of *Stochastic Processes and their Applications*. 1999, pp 187-225.

[3] J. Seidler and I. Vrkoč. *An averaging principle for stochastic evolution equations. I*, volume 115(3) of *Časopis pro pěstování matematiky*. 1990, pp240-263.

[4] B. Maslowski, J. Seidler and I. Vrkoč. *An averaging principle for stochastic evolution equations. II*, volume 116(2) of *Mathematica Bohemica*. 1991, pp191-224.