Numerical evaluation of ruin probabilities in heavy-tailed risk models is an important and challenging problem. In this presentation, we construct very accurate approximations of the ruin probability that capture the tail behavior of the exact ruin probability and provide a small relative error. Motivated by statistical analysis, we assume that the claim sizes are a mixture of a phase-type and a heavy-tailed distribution, and with the aid of perturbation analysis we derive a series expansion for the ruin probability. Our proposed approximations consist of the first two terms of this series expansion, where the first term is a phase-type approximation of the ruin probability. We refer to our approximations collectively as corrected phase-type approximations. In addition, we show how our approximations generalize in the case where claims arrive according to a Markovian Arrival Process. Finally, for a model for which the exact ruin probability can be calculated, we check the accuracy of the corrected phase-type approximations.