STATIONARY SOLUTIONS OF SPATIAL ARMA EQUATIONS

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Consider the d-dimensional spatial ARMA model defined by the equations

$$Y_{\mathbf{t}} - \sum_{\mathbf{n}\in R} \phi_{\mathbf{n}} Y_{\mathbf{t}-\mathbf{n}} = Z_{\mathbf{t}} + \sum_{\mathbf{n}\in S} \theta_{\mathbf{n}} Z_{\mathbf{t}-\mathbf{n}}, \quad \mathbf{t} = (t_1, \dots, t_d) \in \mathbb{Z}^d, \tag{1}$$

where $(Z_{\mathbf{t}})_{\mathbf{t}\in\mathbb{Z}^d}$ is a weak white noise random field and R and S are finite subsets of $\mathbb{Z}^d_+$.

It is known that a sufficient and necessary condition for the existence of a weakly stationary solution of (1) is given by

$$\frac{\Theta(e^{-i\cdot})}{\Phi(e^{-i\cdot})} \in L^2(\mathbb{T}^d), \tag{2}$$

see for example [2].

In this work we study the case, where $(Z_{\mathbf{t}})_{\mathbf{t}\in\mathbb{Z}^d}$ is an i.i.d. random field with not necessarily finite moments, and give necessary and sufficient conditions for the existence of a linear strictly stationary solution of (1), i.e. a random field $(Y_{\mathbf{t}})_{\mathbf{t}\in\mathbb{Z}^d}$ with a representation

$$Y_{\mathbf{t}} = \sum_{\mathbf{n}\in\mathbb{Z}^d} \psi_{\mathbf{n}} Z_{\mathbf{t}-\mathbf{n}}, \quad (\psi_{\mathbf{n}})_{\mathbf{n}\in\mathbb{Z}^d}, \quad \mathbf{t} \in \mathbb{Z}^d, \tag{3}$$

where (3) converges almost surely absolutely. A characterization of the almost sure absolute convergence of (3) is also specified, in terms of log-moment conditions and zeros of the autoregressive polynomial.

Furthermore we study sufficient and necessary conditions for the existence of causal solutions of (1), i.e. a strictly stationary random field $(Y_{\mathbf{t}})_{\mathbf{t}\in\mathbb{Z}^d}$ for which each $Y_{\mathbf{t}}$ is measurable with respect to $\sigma(Z_{\mathbf{s}} : \mathbf{s} \leq \mathbf{t})$. The results generalize the work of [1], who considered $d = 1$.

References

[1] Peter J. Brockwell and Alexander Lindner (2010): *Strictly stationary solutions of autoregressive moving average equations.* Biometrika 97, 765-772.

[2] Murray Rosenblatt. *Gaussian and Non-Gaussian Linear Time Series and Random Fields.* 1st edition, Berlin, Springer, 2000.