A TRANSIENCE CONDITION FOR A CLASS OF ONE-DIMENSIONAL SYMMETRIC LÉVY PROCESSES

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An $\mathbf{R}^d$-valued, $d \geq 1$, Lévy process $\{L_t\}_{t\geq 0}$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is said to be *transient* if $\lim_{t\to\infty} |L_t| = \infty$ $\mathbb{P}$-a.s. and *recurrent* if $\liminf_{t\to\infty} |L_t| = 0$ $\mathbb{P}$-a.s. It is well known that every Lévy process is either transient or recurrent.

Further, every Lévy process $\{L_t\}_{t\geq 0}$ can be completely and uniquely characterized through the characteristic function of a single random variable L_t, $t > 0$, that is, by the famous *Lévy-Khintchine formula* we have

$$\mathbb{E}[\exp\{i\langle \xi, L_t\rangle\}] = \exp\{-t\psi(\xi)\} \quad \text{for all } t \geq 0,$$

where

$$\psi(\xi) = i\langle \xi, b\rangle + \frac{1}{2}\langle \xi, c\xi\rangle + \int_{\mathbf{R}^d} \left(1 - \exp\{i\langle \xi, y\rangle\} + i\langle \xi, y\rangle 1_{\{|y|\leq 1\}}(y)\right) \nu(dy).$$

The characterization of the transience and recurrence property in terms of the characteristic exponent $\psi(\xi)$ is given by the well-known *Chung-Fuchs criterion*: A Lévy process $\{L_t\}_{t\geq 0}$ is transient if and only if

$$\int_{\{|\xi|<a\}} \mathrm{Re}\left(\frac{1}{\psi(\xi)}\right) d\xi < \infty \quad \text{for some } a > 0.$$

In many cases this criterion is not applicable, that is, it is not always easy to compute the above integral. In this talk, we present a transience condition for a class of one-dimensional symmetric Lévy processes in terms of the Lévy measure $\nu(dy)$.

Theorem 1. *Let $\{L_t\}_{t\geq 0}$ be a one-dimensional symmetric Lévy process with the Lévy measure $\nu(dy) = f(y)dy$ or $\nu(n) = p_n$, where $f(y)$ is such that $f(y) > 0$ a.e. and $\{p_n\}_{n\geq 1}$ is such that $p_n > 0$ for all $n \geq 1$. Then, $\{L_t\}_{t\geq 0}$ is transient if*

$$\int_1^\infty \frac{dy}{y^3 f(y)} < \infty \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{1}{n^3 p_n} < \infty.$$

As a simple consequence of Theorem 1 we get a new proof for the transience property of one-dimensional symmetric stable Lévy processes.

Corollary 1. *A one-dimensional symmetric α-stable Lévy process is transient if $\alpha < 1$.*

Also, let us remark that the analogous transience condition holds for one-dimensional symmetric random walks.

References

[1] James P. Hobert and Jason Schweinsberg. *Conditions for recurrence and transience of a Markov chain on $\mathbb{Z}^+$ and estimation of a geometric success probability.* The Annals of Statistics **30**(4):1214–1223, 2002.

[2] Terry Lyons. *A simple criterion for transience of a reversible Markov chain.* The Annals of Probability **11**(2):393–402, 1983.

[3] Nikola Sandrić. *A transience condition for a class of one-dimensional symmetric Lévy processes.* Preprint, 2013.

[4] Ken-iti Sato. *Lévy processes and infinitely divisible distributions*, volume 68 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.