# AN ITō FORMULA FOR CONVOLUTED LÉVY PROCESSES

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In this talk we present an Itō-type formula for convoluted Lévy processes. Here a convoluted Lévy process $(M(t))_{t\in\mathbb{R}}$ is defined via the integral representation

$$M(t) = \int_{\mathbb{R}} f(t,s)\ L(ds),$$

where $f$ is some kernel function and $L$ is a Lévy process with jump measure $N(dx, ds)$.

In the spirit of [1] and [2] the main result presented in this talk is the following change of variable formula:

**Theorem 1.** *Let $(M(t))_{t\in\mathbb{R}}$ be a convoluted Lévy process with an appropriate kernel function $f$. Furthermore let $G \in C^2(\mathbb{R})$ with $G, G'$ and $G''$ in $\mathscr{L}^1(\mathbb{R})$ and $0 < T < \infty$. Then the Itō-type formula*

$$\begin{aligned} G(M(T)) &= G(0) + \sigma^2 \int_0^T G''(M(t-)) \left( \frac{f(t,t)^2}{2} + \int_{-\infty}^t f(t,s) \frac{\partial}{\partial t} f(t,s) ds \right) dt \\ &+ \int_0^T G'(M(t-)) M^{\diamond}(dt) \\ &+ \sum_{0<t\leq T} [G(M(t)) - G(M(t-)) - G'(M(t-))\Delta M(t)] \\ &+ \int_0^T \int_{-\infty}^t \int_{\mathbb{R}_0} \left(G'\left(M(t-) + x f(t,s)\right) - G'(M(t-))\right) x \frac{\partial}{\partial t} f(t,s)\ N^{\diamond}(dx, ds) dt \end{aligned}$$

*holds if all the terms exist in $\mathscr{L}^2(\Omega)$.*

The $^{\diamond}$-integrals appearing in Theorem 1 are understood as Skorokhod integrals, defined via the $S$-transform. The proof uses an injectivity argument for the $S$-transform as derived in [3]. In particular, Theorem 1 holds for a general class of fractional Lévy processes.

## References

[1] Christian Bender, *An S-transform approach to integration with respect to a fractional Brownian motion*, Bernoulli **9** (2003), no. 6, 955–983. MR 2046814 (2005d:60056)

[2] Christian Bender and Tina Marquardt, *Stochastic calculus for convoluted Lévy processes*, Bernoulli **14** (2008), no. 2, 499–518. MR 2544099 (2010k:60199)

[3] Yuh-Jia Lee and Hsin-Hung Shih, *The Segal-Bargmann transform for Lévy functionals*, J. Funct. Anal. **168** (1999), no. 1, 46–83. MR 1717851 (2001c:60107)