RECIPROCAL CLASSES OF JUMP PROCESSES

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In this talk we present our first results on the reciprocal class associated with a pure jump Lévy process $\mathbb{P}$, that is the set of all probability measures that have same bridges as $\mathbb{P}$. We make explicit the connection with the notion of h-transform and introduce reciprocal invariants that are characteristics of the class. We will highlight how the structure of the support of the jump measure plays a key role studying the basic case of compound Poisson processes.

We consider two probability measures $\mathbb{P}$ and $\tilde{\mathbb{P}}$ and make the following assumptions:

- $\mathbb{P}$, $\tilde{\mathbb{P}}$ are the distributions of two pure jump Lévy processes on the time interval $[0, 1]$ with finite activity and initial distribution δ_0.
- The associated jump measures ν and $\tilde{\nu}$ are such that $\tilde{\nu} << \nu$

We recall the notion h-transform. Intuitively, an h transform, share the same bridges with the original probability.

Definition 1. $\tilde{\mathbb{P}}$ is an h-transform of $\mathbb{P}$ if $\tilde{\mathbb{P}} << \mathbb{P}$ and the density is X_1-measurable (i.e. the density is measurable w.r.t. to the state of the process at time 1)

Theorem 1. *Let $\mathbb{P}$ and $\tilde{\mathbb{P}}$ as above. Then $\tilde{\mathbb{P}}$ is an h-transform of $\mathbb{P}$ if and only if, there exists a function ψ (potential function) such that:*

$$\sum_{j=1}^{n} g(q^j) = \psi(q^1 + ... + q^n) - \psi(0) \ \nu \times ... \times \nu \ a.e. \ \forall n$$

where $g = \log \frac{d\tilde{\nu}}{d\nu}$.

We give two examples, showing how the support of the jump measure is important in the analysis of the bridges.

Example 1. Two compound Poisson processes with with jumps in $\{-1, +1\}$ have the same bridges if and only if $\nu(1)\nu(-1) = \tilde{\nu}(1)\tilde{\nu}(-1)$. This has an interpretation as a loop condition on the intensities.

Example 2. Any two compound Poisson processes with with jumps in $\left\{1, \sqrt{2}\right\}$ have the same bridges.

References

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