Corrected phase-type approximations of heavy-tailed risk models using perturbation analysis

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Cramér-Lundberg model:

- λ: poisson arrival rate (N_t)
- U_i: i.i.d. claim sizes (F)
- $\rho = \lambda \mathbb{E} U < 1$: claims per unit rate
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Ruin probability:

$$\psi(u) = \mathbb{P}(M > u).$$

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$$\psi(u) = 1 - (1-\rho)\sum_{n=0}^{\infty}\rho^n (F^e)^{*n}(u)$$

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$$m(s) = \frac{1-\rho}{1-\rho\upsilon^e(s)}.$$

Introduction

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Solution: approximations

Phase-type approximations

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1. High accuracy
2. Computationally tractable
3. Error bounds
4. Big relative error at the tail

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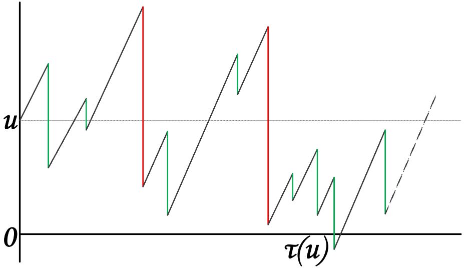
New idea: Separate the claim sizes



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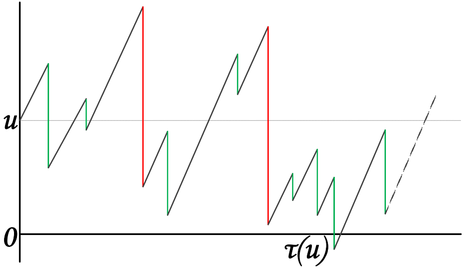

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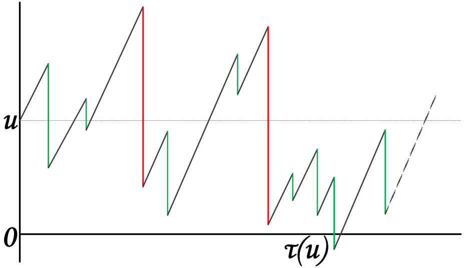

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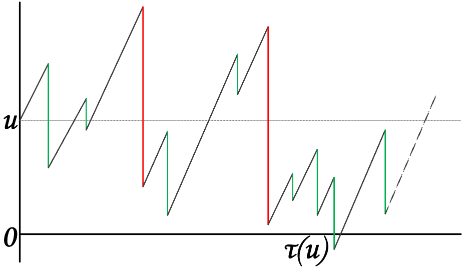

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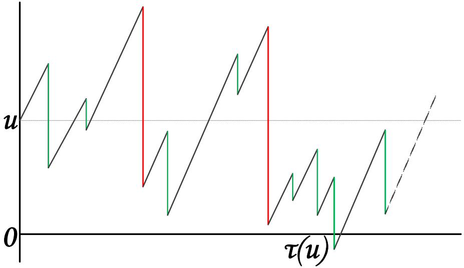

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Claim size distribution

$$F(x) = (1 - \epsilon)F_p(x) + \epsilon F_h(x).$$

Laplace Transform of Pollaczek-Khinchine formula

$$m_\epsilon(s) = \frac{1 - (1-\epsilon)\delta - \epsilon\theta}{1 - (1-\epsilon)\delta\beta^e(s) - \epsilon\theta\gamma^e(s)}.$$

Mixture model

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Notation:

- Claim rate: $\rho_\epsilon = (1-\epsilon)\underbrace{\lambda\mathbb{E}B}_{=\delta} + \epsilon\underbrace{\lambda\mathbb{E}C}_{=\theta} < 1.$
- $\beta^e(s)$: LT of stationary-excess phase-type claim sizes,
- $\gamma^e(s)$: LT of stationary-excess heavy-tailed claim sizes.

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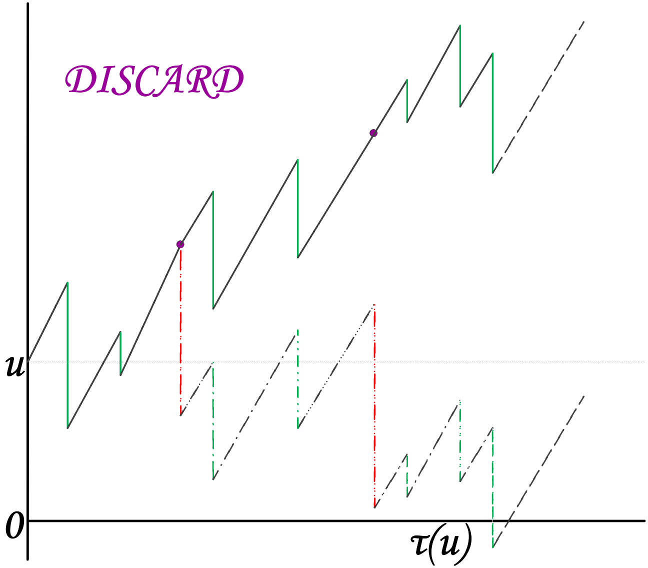
Related work: Perturbed risk processes of Huzak et al. (2004)

Handling the heavy-tailed claims



Discard series expansion of the ruin probability

$$m_\epsilon^\bullet(s) = \frac{1-(1-\epsilon)\delta}{1-(1-\epsilon)\delta\beta^e(s)} \qquad \text{(discard PH)}$$

$$\Rightarrow$$

$$\begin{aligned} m_\epsilon(s) =& m_\epsilon^\bullet(s) \\ &+ \sum_{n=1}^{\infty}\left(\frac{\epsilon\theta}{1-\delta+\epsilon\delta}\right)^n \left(m_\epsilon^\bullet(s)\right)^{n+1}\left(\gamma^e(s)\right)^n \\ &- \sum_{n=1}^{\infty}\left(\frac{\epsilon\theta}{1-\delta+\epsilon\delta}\right)^n \left(m_\epsilon^\bullet(s)\right)^{n}\left(\gamma^e(s)\right)^{n-1} \end{aligned}$$

Discard series expansion of the ruin probability

Discard series expansion

If $\psi_\epsilon^\bullet(u)$ is the discard PH approximation of $\psi_\epsilon(u)$, and $L_n(u) = \mathbb{P}(M_{\epsilon,0}^\bullet + M_{\epsilon,1}^\bullet + \cdots + M_{\epsilon,n}^\bullet + C_1^e + \cdots + C_n^e > u)$, then

$$\psi_\epsilon(u) = \psi_\epsilon^\bullet(u) + \sum_{n=1}^{\infty} \left(\frac{\epsilon\theta}{1 - \delta + \epsilon\delta} \right)^n (L_n(u) - L_{n-1}(u)),$$

where $L_0(u) = \psi_\epsilon^\bullet(u)$. A necessary and sufficient condition for convergence $\forall u$ is: $|\epsilon\theta| < |1 - \delta + \epsilon\delta|$.

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Corrected discard approximation

$$\tilde{\psi}_{d,\epsilon}(u) = \psi_\epsilon^\bullet(u) + \frac{\epsilon\theta}{1 - \delta + \epsilon\delta} \left[\mathbb{P}(M_{\epsilon,0}^\bullet + M_{\epsilon,1}^\bullet + C_1^e > u) - \mathbb{P}(M_{\epsilon,0}^\bullet > u) \right]$$

Properties of corrected discard approximation

1. Error bounds

$$\left(\frac{\epsilon\theta}{1-\delta+\epsilon\delta}\right)^2 \left(L_2(u) - L_1(u)\right) \quad \leq \psi_\epsilon(u) - \tilde{\psi}_{d,\epsilon}(u) \quad \leq \left(\frac{\epsilon\theta}{1-\delta+\epsilon\delta}\right)^2$$

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Remarks:

- Positive error of $O(\epsilon^2)$, and

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Remarks:

- Positive error of $O(\epsilon^2)$, and
- corrected discard always underestimates the exact ruin probability.

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- tail of corrected discard below the tail of exact ruin probability $\left(\psi_\epsilon(u) \sim \frac{\epsilon\theta}{1-\delta+\epsilon\delta-\epsilon\theta}\mathbb{P}(C^e > u)\right)$.

Properties of corrected discard approximation

3. Relative error

There exists an $\eta > 0$, such that for all $\epsilon < \eta$, the relative error $R_d(u)$ of the discard approximation at the point u can be bounded by

$$R_d(u) \leq \frac{\epsilon\theta}{1-\delta+\epsilon\delta} H(u) + \epsilon^2 K,$$

with $H(u) = \left(\frac{P(M^{\bullet}_{\epsilon,0}+M^{\bullet}_{\epsilon,1}+M^{\bullet}_{\epsilon,2}+C^e_1+C^e_2>u)}{P(M^{\bullet}_{\epsilon,0}+M^{\bullet}_{\epsilon,1}+C^e_1>u)} - 1 \right)$ and K a finite constant.

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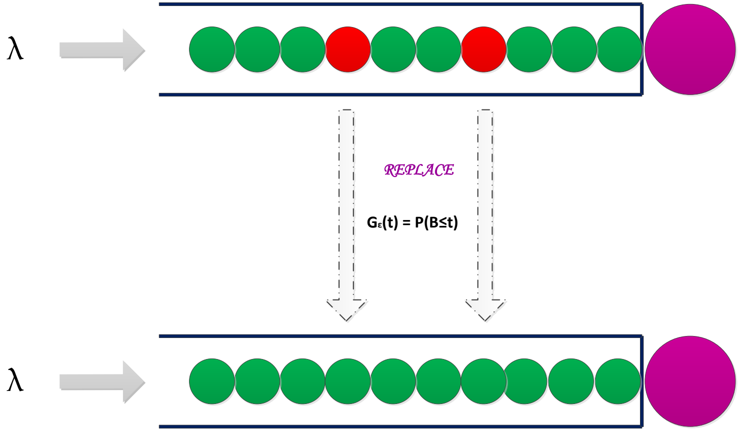
Alternative way to handle the heavy-tailed claims



Corrected discard approximation and properties

Corrected discard approximation

$$\tilde{\psi}_{r,\epsilon}(u) := \psi_0(u) + \frac{\epsilon\theta}{1-\delta}\left(\mathbb{P}(M_{0,0} + M_{0,1} + C_1^e > u) - \mathbb{P}(M_{0,0} > u)\right)$$
$$- \frac{\epsilon\delta}{1-\delta}\left(\mathbb{P}(M_{0,0} + M_{0,1} + B_1^e > u) - \mathbb{P}(M_{0,0} > u)\right)$$

Properties:

1. Error bound

$$|\psi_\epsilon(u) - \tilde{\psi}_{r,\epsilon}(u)| \leq \left(\frac{\epsilon}{1-\delta}\right)^2 (\delta+\theta)^2 \frac{1-\delta}{1-\delta-\epsilon(\delta+\theta)}, \forall \epsilon < |1-\delta|/(\delta+\theta).$$

Remark: error of $O(\epsilon^2)$.

Properties of corrected replace approximation

2. Tail behavior

$$\tilde{\psi}_{r,\epsilon}(u) \sim \frac{\epsilon\theta}{1-\delta}\mathbb{P}(C^e > u), \text{ when } C^e \in \mathcal{S}.$$

Remarks:

- tail of corrected replace always above tail of corrected discard, and
- tail of corrected replace below the exact ruin probability when $\delta < \theta$.

3. Relative error at the tail

$$|R_r(u)| = \left|1 - \frac{\tilde{\psi}_{r,\epsilon}(u)}{\psi_\epsilon(u)}\right| \to \left|\frac{\epsilon(\theta-\delta)}{1-\delta}\right|, \quad \text{as} \quad u \to \infty.$$

Remarks:

- it goes asymptotically to zero when $\mathbb{E}B = \mathbb{E}C$, and
- relative error at tail of $O(\epsilon)$.

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Numerical results

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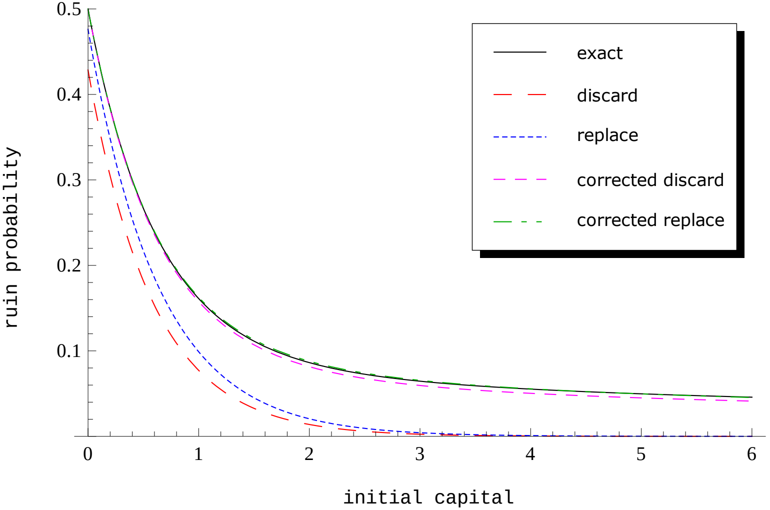
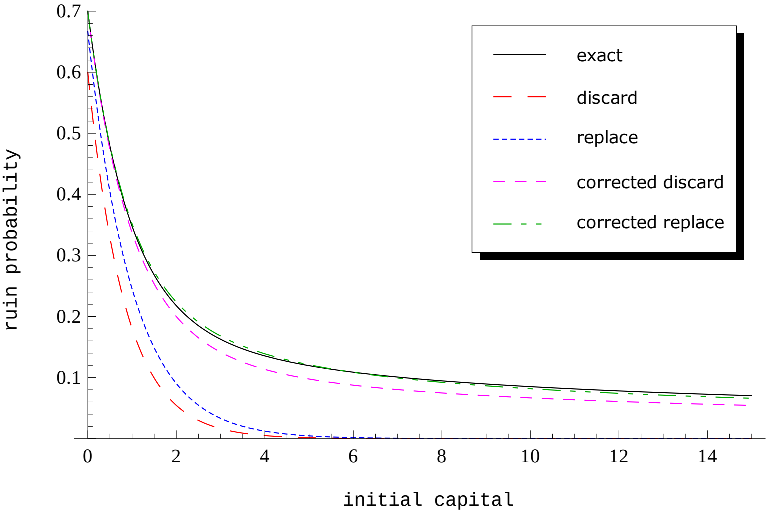
Remark: The "worst case scenario" for ϵ is the "best case scenario" for the improvement we achieve with the corrected phase-type approximations.

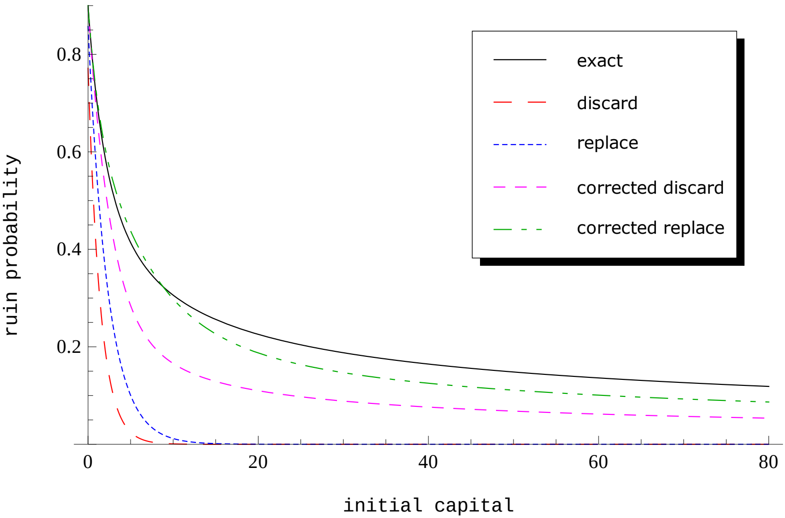
Approximations when $\epsilon = 0.1$ and $\rho_\epsilon = 0.5$.



Approximations when $\epsilon = 0.1$ and $\rho_\epsilon = 0.7$.



Approximations when $\epsilon = 0.1$ and $\rho_\epsilon = 0.9$.



Approximations when $\epsilon = 0.001$ and $\rho_\epsilon = 0.5$.

t	exact	discard	replace	cor.discard	cor.replace
0	0.50000000	0.49925037	0.49975012	0.50000000	0.50000000
1	0.11211000	0.11114757	0.11142576	0.11210955	0.11211017
2	0.02557910	0.02474466	0.02484381	0.02557847	0.02557930
3	0.00621454	0.00550887	0.00553925	0.00621386	0.00621466
4	0.00184042	0.00122643	0.00123504	0.00183975	0.00184047
5	0.00082276	0.00027304	0.00027536	0.00082212	0.00082275
6	0.00056334	0.00006078	0.00006139	0.00056273	0.00056329
7	0.00047969	0.00001353	0.00001368	0.00047910	0.00047962
8	0.00043993	3.01×10^{-6}	3.05×10^{-6}	0.00043937	0.00043985
9	0.00041336	6.70×10^{-7}	6.80×10^{-7}	0.00041284	0.00041329
10	0.00039235	1.49×10^{-7}	1.51×10^{-7}	0.00039183	0.00039225

Table : Exact ruin probability with phase-type and corrected phase-type approximations

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- no requirement of finite higher-moments for the service times (only finite mean),
- better numerical estimates from corrected replace approximation,
- nature of the error of corrected discard gives better theoretical results.

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- dense in the class of marked point processes,
- important in telecommunication systems, etc.

Corrected approximations

Corrected discard (replace)

$$\mathbb{P}(\hat{V}_\epsilon > t) := \mathbb{P}(V_\epsilon^\bullet > t) + \epsilon \frac{1}{\mathbf{u}_\epsilon^\bullet \mathbf{e}} \Bigg(\alpha' \mathbb{P}(V_\epsilon^\bullet > t) + \sum_{j=1}^{mr} \alpha_j' \mathbb{P}(V_\epsilon^\bullet + E_{y_j} > t)$$

$$+ \sum_{i=2}^{N} \delta_i' \mathbb{P}(t < V_\epsilon^\bullet < t + E_{s_i})$$

$$+ \beta \big(\mu_p \mathbb{P}(V_\epsilon^\bullet + B^e > t) - \mu_h \mathbb{P}(V_\epsilon^\bullet + C^e > t) \big)$$

$$+ \sum_{j=1}^{mr} \beta_j \big(\mu_p \mathbb{P}(V_\epsilon^\bullet + B^e + E_{y_j} > t) - \mu_h \mathbb{P}(V_\epsilon^\bullet + C^e + E_{y_j} > t) \big)$$

$$+ \sum_{i=2}^{N} \eta_i \big(\mu_p \mathbb{P}(t < V_\epsilon^\bullet + B^e < t + E_{s_i}) - \mu_h \mathbb{P}(t < V_\epsilon^\bullet + C^e < t + E_{s_i}) \big)$$

$$+ \gamma \big(\mu_p \mathbb{P}(V_\epsilon^\bullet + V_\epsilon^\bullet{}' + B^e > t) - \mu_h \mathbb{P}(V_\epsilon^\bullet + V_\epsilon^\bullet{}' + C^e > t) \big)$$

$$+ \sum_{j=1}^{mr} \gamma_j \big(\mu_p \mathbb{P}(V_\epsilon^\bullet + V_\epsilon^\bullet{}' + B^e + E_{y_j} > t) - \mu_h \mathbb{P}(V_\epsilon^\bullet + V_\epsilon^\bullet{}' + C^e + E_{y_j} > t) \big)$$

$$+ \sum_{i=2}^{N} \theta_i \big(\mu_p \mathbb{P}(t < V_\epsilon^\bullet + V_\epsilon^\bullet{}' + B^e < t + E_{s_i}) - \mu_h \mathbb{P}(t < V_\epsilon^\bullet + V_\epsilon^\bullet{}' + C^e < t + E_{s_i}) \big) \Bigg).$$

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Thank you for your attention