

On symmetries of singular implicit ODEs

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Abstract

We study implicit ODEs, cubic in derivative, with infinitesimal symmetry at singular points. Cartan showed that even at regular points the existence of nontrivial symmetry imposes restrictions on the ODE. Namely, this algebra has the maximal possible dimension 3 iff the web of solutions is flat. For cubic ODEs with flat 3-web of solutions we establish sufficient conditions for the existence of non-trivial symmetries at singular points and show that under natural assumptions such a symmetry is semi-simple, i.e. is a scaling in some coordinates. We use this symmetry to find first integrals of the ODE.

Key words: hexagonal 3-web, infinitesimal symmetries, Chern connection, implicit ODE.

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