Tame Bi-Lipschitz contact equivalence of tame functions

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We want to classify continuous functions germs within some prescribed *tame* algebra (e.g. real analytic, continuous sub-analytic, etc...) under the corresponding *tame bi-Lipschitz contact equivalence.* (*TbLC-equivalence* for short), defined as follows:

Two continuous functions germs $f, g : (\mathbb{R}^n, \mathbf{0}) \to (\mathbb{R}, 0)$ are *TbLC-equivalent* if there exists a tame bi-Lipschitz homeomorphism germ $H : (\mathbb{R}^n, \mathbf{0}) \to (\mathbb{R}^n, \mathbf{0})$ and positive constants A, B such that the following alternative holds true:

$$\text{either} \quad Af \leq \ g \circ H \ \ \leq Bf,$$

$$\text{or} \quad Af \leq -g \circ H \leq Bf\,.$$

From the infinitesimal point of view, the inequalities above means that the functions f, g (or $f, -g$) have the *same asymptotics at the origin.*

In the plane case ($n = 2$) and when working in any à-priori given polynomially bounded o-minimal structure $\mathcal{M}$ expanding the field $\mathbb{R}$ (e.g. semi-algebraic), to any continuous function germ definable (in $\mathcal{M}$), we associate a finite combinatorial object, called *the Hölder Diagram of the function at the origin,* consisting of a definable triangulation of the germ $(\mathbb{R}^2, 0)$ and some finite data (an affine function, and two signs) on each triangle (= 2-simplex).

Our main result states that: *two Lipschitz and definable (in $\mathcal{M}$) plane function germs are TbLC-equivalent if and only if they have the same Hölder Diagram.*

In the talk I will develop on and makes sense of the notion of "(same) asymptotics at the origin".

I will introduce the new notion of *width of* (a tame) *curve germ* with respect to a function germ, which will reveal key to completely understand the "asymptotics" of the function at the origin. This is best summarized in the notion of *minimal Hölder data,* (though we have to prove its existence and uniqueness in some meaningful sense).

Eventually, the *Hölder Diagram* is built from the minimal Hölder data by just adding the information necessary to distinguish the (finitely many) different TbLC-equivalence classes with a given minimal Hölder data.

This is a joint work with L. Birbrair (UFC), A. Fernandes (UFC) and A. Gabrielov (Purdue).