Mappings from $\mathbb{R}^3$ to $\mathbb{R}^3$, and swallowtails

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Let M be an oriented 3-manifold. For a residual set of mappings $f \in C^\infty(M, \mathbb{R}^3)$, the 3-jet $j^3 f$ intersect transversely the Boardman strata S_1, $S_{1,1}$, $S_{1,1,1}$ in $J^3(M, \mathbb{R}^3)$. If that is the case then $S_{1,1,1}(f)$ is a discrete subset of M.

If $j^3 f$ is also transversal to S_{1_3} then $S_{1,1,1}(f) = S_{1_3}(f)$, and for $p \in S_{1,1,1}(f)$ there exists a well-oriented coordinate system x, y, z centered at p, and a coordinate system centered at $f(p)$ such that f has the form $f_\pm(x, y, z) = (\pm xy + x^2 z + x^4, y, z)$, so one may associate with $p \in S_{1,1,1}(f)$ an index $\in \{\pm 1\}$. Another geometric definition of the sign associated with a swallowtail was presented in [2].

We give a definition of a simple swallowtail point $p \in S_{1,1,1}(f)$, and define its index $I(f, p)$. We shall show that

- if p is a simple swallowtail then $j^3 f$ intersect transversely S_{1_3} at p,
- $I(f, p) = +1$ (resp. -1) if and only if f has the form f_+ (resp. f_-).

If every $p \in S_{1,1,1}(f)$ is a simple swallowtail and $S_{1,1,1}(f)$ is finite, then numbers $\#S_{1,1,1}(f)$, $\#S^\pm_{1,1,1}(f) = \#\{p \in S_{1,1,1}(f) \mid I(f, p) = \pm 1\}$, where $\#$ denotes the number of points, are important invariants associated with f. (The first-order invariants of stable mappings in $C^\infty(M, \mathbb{R}^3)$ were classified in [1, 2]) We shall show how to compute the numbers $\#S_{1,1,1}(f)$, $\#S^\pm_{1,1,1}(f)$ in the case where $f : \mathbb{R}^3 \to \mathbb{R}^3$ is a polynomial mapping.

References

[1] M. Carmen Romero-Fuster, and R. Oset Sinha, *First-order local invariants of stable maps from 3-manifolds to* $\mathbb{R}^3$, Michigan Math. J., 61 (2) (2012), 385-414.

[2] V. Goryunov, Local invariants of maps between 3-manofolds, Journal of Topology, first published online May 21, 2013 doi:10.1112/jtopol/jtt015