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Hypercontractivity and long time behaviour in nonautonomous Kolmogorov equations

Joint work with Alessandra Lunardi and Luca Lorenzi.

We consider nonautonomous Cauchy problems,

$$\begin{cases} D_t u(t,x) = \mathcal{A}(t)u(t,x), & (t,x) \in (s,+\infty)\times\mathbb{R}^d, \\ u(s,x) = f(x), & x\in\mathbb{R}^d, \end{cases}$$

where $\{\mathcal{A}(t)\}_{t\in I}$ is a family of second order differential operators,

$$(\mathcal{A}(t)\zeta)(x) = \mathrm{Tr}(Q(t)D^2\zeta(x)) + \langle b(t,x), \nabla\zeta(x)\rangle,$$

with smooth enough coefficients $Q = [q_{ij}]_{i,j=1,\dots,d}$ and $b = (b_1,\dots,b_d)$, (possibly unbounded), defined in I and $I\times\mathbb{R}^d$, respectively, where I is an open right halfline and $s\in I$.
It is well known that the usual L^p spaces with respect to the Lebesgue measure dx are not a natural setting for elliptic and parabolic operators with unbounded coefficients, unless quite strong growth assumptions are imposed on their coefficients. Much better settings are L^p spaces with respect to the so called evolution systems of measures $\{\mu_t : t\in I\}$ associated to the evolution operator $G(t,s)$, i.e. a family of Borel probability measures in $\mathbb{R}^d$ satisfying

$$\int_{\mathbb{R}^d} G(t,s)f d\mu_t = \int_{\mathbb{R}^d} f d\mu_s =: m_s f, \qquad t>s\in I,\ \ f\in C_b(\mathbb{R}^d).$$

We prove hypercontractivity results in the spaces $L^p(\mathbb{R}^d,\mu_t)$ and we study the asymptotic behavior of $G(t,s)$ as $t\to+\infty$.
The starting point of our analysis is the proof of the logarithmic Sobolev inequality for the measures μ_t, in the form

$$\int_{\mathbb{R}^d} |f|^p\log|f|\,d\mu_t \le \frac{1}{p}\left(\int_{\mathbb{R}^d}|f|^p d\mu_t\right)\log\left(\int_{\mathbb{R}^d}|f|^p d\mu_t\right) + pC\int_{\mathbb{R}^d}|f|^{p-2}|\nabla f|^2\chi_{\{f\neq 0\}}d\mu_t,$$

for any $t\in I$, any $p\in(1,+\infty)$ and some positive constant C, independent of $f\in C_b^1(\mathbb{R}^d)$, t and p.

The logarithmic Sobolev inequality has a crucial role in the proof of the hypercontractivity results in the spaces $L^p(\mathbb{R}^d,\mu_t)$ which, together with the Poincaré inequality, allow us to compare the asymptotic behavior of $\|G(t,s)f - m_s f\|_{L^p(\mathbb{R}^d,\mu_t)}$ and $\|\,|\nabla_x G(t,s)f|\,\|_{L^p(\mathbb{R}^d,\mu_t)}$.

References

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