**Jacek Dziubański**
Instytut Matematyczny, Uniwersytet Wrocławski
**Jacek Zienkiewicz**
Instytut Matematyczny, Uniwersytet Wrocławski

# On isomorphisms of Hardy spaces for certain Schrödinger operators

Let $\{K_t\}_{t>0}$ be the semigroup of linear operators on $\mathbb{R}^d$, $d \geq 3$, generated by a Schrödinger operator, where $V \geq 0$. We say that an $L^1$-function $f$ belongs to the Hardy space $H^1_L$ associated with $L$ if the maximal function

$$\mathcal{M}f(x) = \sup_{t>0} |K_t f(x)|$$

belongs to $L^1(\mathbb{R}^d)$.

We shall prove that the following two conditions are equivalent:

(1) there is an $L$-harmonic function $w$, $0 < \delta \leq w(x) \leq C$, such that the mapping

$$H^1_L \ni f \mapsto wf \in H^1(\mathbb{R}^d)$$

is an isomorphism of the Hardy space $H^1_L$ and the classical Hardy space $H^1(\mathbb{R}^d)$;

(2) the global Kato norm $\|V\|_{\mathcal{K}}$ is finite, where

$$\|V\|_{\mathcal{K}} = \sup_{x\in\mathbb{R}^d} \int_{\mathbb{R}^d} |x-y|^{2-d} V(y)\, dy.$$

The second result states that in this case the operator $(-\Delta)^{1/2} L^{-1/2}$ is an another isomorphism of the spaces $H^1_L$ and $H^1(\mathbb{R}^d)$.

As corollaries we obtain that the space $H^1_L$ admits:

(3) atomic decomposition with atoms satisfying the support condition $\operatorname{supp} a \subset B$ (for a certain ball $B$), the size condition $\|a\|_{L^\infty} \leq |B|^{-1}$, and the cancellation condition $\int a(x)w(x)dx = 0$

(4) characterization by the Riesz transforms $R_j = \partial_{x_j} L^{-1/2}$.

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