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Discrete Coagulation-Fragmentation Equations

Coagulation and fragmentation processes arise in a number of areas of pure and applied science. Examples include colloidal aggregation, blood clotting and polymer science. The usual starting point when developing a mathematical model of such processes is to regard the system under consideration as one consisting of a large number of clusters that can coagulate to form larger clusters or fragment into a number of smaller clusters. Under the assumption that each cluster of size n consists of n identical fundamental units (monomers), we obtain a discrete model of coagulation-fragmentation which takes the form of an infinite system of ordinary differential equations.

In this talk, the associated initial-value problem for this infinite-dimensional system will be expressed as a semi-linear abstract Cauchy problem, posed in a physically relevant Banach space. Perturbation results from the theory of semigroups of operators will be used to establish the existence and uniqueness of globally-defined, strongly differentiable, non-negative solutions for uniformly bounded coagulation rates but with minimal restrictions placed on the fragmentation rates.

In one specific case of a pure fragmentation process, in which no coagulation occurs, an interesting phenomenon arises due to the existence of an explicit solution, which despite satisfying homogeneous initial conditions in a pointwise manner, appears to emanate from an initial state that has unit mass. This apparent paradox will be explained in a satisfactory manner by using the theory of Sobolev towers.

A couple of recent extensions of the existence/uniqueness results discussed in the first part of the talk will also be mentioned briefly. The first is concerned with a system of clusters which are distinguished, not only by size, but also by shape. The second, due to Jacek Banasiak, employs theory associated with analytic semigroups to relax the assumption that the coagulation rates are uniformly bounded.

This is joint work with Adam McBride, Louise Smith and Matthias Langer.

References

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