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Semigroups and the maximum principle for structured populations with diffusion

We study a size-structured model which describes the dynamics of one population with growth, diffusion, reproduction and mortality rates, i.e.

$$\begin{aligned} u_t(t,s) &= \left(d(s)u_s(t,s)\right)_s - \left(\gamma(s)u(t,s)\right)_s - \mu(s)u(t,s) \\ &\quad + \int_0^m \beta(s,y)u(t,y)\,dy + g(t,s), \quad s \in (0,m) \end{aligned}$$

with linear Feller boundary conditions

$$\begin{aligned} \left[\left(d(s)u_s(t,s)\right)_s\right]_{s=0} - b_0 u_s(t,0) + c_0 u(t,0) = 0 \\ \left[\left(d(s)u_s(t,s)\right)_s\right]_{s=m} + b_m u_s(t,m) + c_m u(t,m) = 0 \end{aligned}$$

and the initial condition

$$u(0,s) = \omega(s), \quad \omega(s) \geq 0.$$

The present paper raises and develops the ideas found in [1], where the autors showed that the size structured model with certain boundary conditions is governed by a positive quasicontractive semigroup on a biologically relevant state space. The advantage of the semigroup approach is that it enables the description of population processes as dynamical systems in the state space. It seems that positivity of solutions is technical and tedious in their semigroup setting, whereas our approach is straightforward. The asymptotic behaviour of solutions is deduced in our study simply by means of the maximum principle.

The aim of this article is to provide more precise attempts to asymptotic analysis in a Hilbert space where one can recognize a finite dimensional subspace attracting some solutions. We prove a weak maximum principle for structured populations models with dynamic boundary conditions. We establish existence and positivity of solutions of these models and investigate the asymptotic behaviour of solutions. In particular, we analyse so called *size profile*.

References

[1] A. Farkas, P. Hinow, *Physiologically structured populations with diffusion and dynamic boundary conditions*, Math. Biosci. Eng. 8 (2) (2011), 503–513.