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An Optimal Control over Solutions of the Initial-Finish Problem for One Class of Linear Sobolev Type Equations

A lot of initial-boundary value problems for the equations and the systems of equations not resolved with respect to time derivative are considered in the framework of abstract Sobolev type equations that make up the vast field of non-classical equations of mathematical physics. Let $\mathfrak{X}, \mathfrak{Y}$ and $\mathfrak{U}$ be the Hilbert spaces. The operators $L \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$, $M \in \mathcal{C}l(\mathfrak{X}; \mathfrak{Y})$ and (L, p)-sectorial [1], $p \in \{0\} \cup \mathbb{N}$ and $B \in \mathcal{L}(\mathfrak{U}; \mathfrak{Y})$. Consider the equation

$$L\dot{x} = Mx + y + Bu, \quad \ker L \neq \{0\}. \tag{1}$$

Here functions $y : (0, \tau) \subset \mathbb{R}_+ \to \mathfrak{Y}$, $u : (0, \tau) \subset \mathbb{R}_+ \to \mathfrak{U}\,(\tau < \infty)$. The theory of degenerate semigroups of operators [1] is a suitable mathematical tool for the study of such problems. We consider the initial-finish problem [2], that is, Sobolev type linear equation (1) with the conditions

$$P_{in}(x(0) - x_0) = 0,\ P_{fin}(x(\tau) - x_\tau) = 0. \tag{2}$$

Here $\tau \in \mathbb{R}_+$, x_0, $x_\tau \in \mathfrak{X}$, the operators P_{in}, P_{fin} are the relatively spectral projections acting in the space $\mathfrak{X}$. The initial-finish problem (1), (2) is a natural generalization of the Showalter –Sidorov problem, which is a generalization of the Cauchy problem. The conditions (2) are different from those previously studied in that one projection of the solution is given at the initial moment, and the other is given at the final moment of the considered time period. We are interested in optimal control problem, which is to find such a pair $(\hat{x}, \hat{u}) \in \mathfrak{X} \times \mathfrak{U}_{ad}$, for which the relation

$$J(\hat{x}, \hat{u}) = \inf_{(x,u) \in \mathfrak{X} \times \mathfrak{U}_{ad}} J(x, u), \tag{3}$$

wherein all pairs (x, u) satisfy the problem (1), (2), takes place. Here

$$J(x, u) = \mu \sum_{q=0}^{1} \int_0^\tau \|z^{(q)} - z_0^{(q)}\|_{\mathfrak{Z}}^2 dt + \nu \sum_{q=0}^{k} \int_0^\tau \left\langle N_q u^{(q)}, u^{(q)} \right\rangle_{\mathfrak{U}} dt$$

is a specially constructed cost functional, $u \in \mathfrak{U}_{ad}$ is the control, $\mathfrak{U}_{ad}$ is a closed and convex set in the control space $\mathfrak{U}$. The operators $N_q \in \mathcal{L}(\mathfrak{U})$, $q = 0, 1, \ldots, p+1$ are self-adjoint and positive definite; $z_0 = z_0(t)$ is the desired observation and $\mu, \nu \geq 0$, $\mu + \nu = 1$, $0 \leq k \leq p+1$. Consider the Hilbert space of observations $\mathfrak{Z}$ and the operator $C \in \mathcal{L}(\mathfrak{X}; \mathfrak{Z})$ defining the observation $z(t) = Cx(t)$. Note that if $x \in H^1(X)$, then $z \in H^1(Z)$.

Introduce the following conditions.

The L-spectrum of the operator M be represented in the form

$$\sigma^L(M) = \sigma^L_{fin}(M) \cup \sigma^L_{in}(M), \tag{A1}$$

where $\sigma^L_{fin}(M)$ is contained in a bounded domain $\Omega \subset \mathbb{C}$ with a piecewise smooth boundary γ, and $\gamma \cap \sigma^L(M) = \emptyset$;

$$\mathfrak{X}^0 \oplus \mathfrak{X}^1 = \mathfrak{X} \ (\mathfrak{Y}^0 \oplus \mathfrak{Y}^1 = \mathfrak{Y}); \tag{A2}$$

$$\text{the operator } L_1^{-1} \in \mathcal{L}(\mathfrak{Y}^1; \mathfrak{X}^1) \text{ exists.} \tag{A3}$$

Construct the spaces

$$H^{p+1}(\mathfrak{Y}) = \{v \in L_2(0,\tau;\mathfrak{Y}) : v^{(p+1)} \in L_2(0,\tau;\mathfrak{Y}), p \in \{0\} \cup \mathbb{N}\}.$$

The space $H^{p+1}(\mathfrak{Y})$ is Hilbert, because we deal with the Hilbert space $\mathfrak{Y}$ endowed with the inner product

$$[v, w] = \sum_{q=0}^{p+1} \int_0^\tau \left\langle v^{(q)}, w^{(q)} \right\rangle_{\mathfrak{Y}} dt.$$

Theorem 1. [3] Let the operator M be (L,p)-sectorial, $p \in \{0\} \cup \mathbb{N}$ and conditions (A1)–(A3) are fulfilled. Then, for all $y \in H^{p+1}(\mathfrak{Y})$, $x_0, x_\tau \in \mathfrak{X}$ there exists a unique optimal control over solutions of the problem (1), (2).

References

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