Semigroups of Operators: Theory and Applications

Book of abstracts

Będlewo, Poland, October 6 — 11, 2013

Contents

Conference schedule

Monday

8^{00} Breakfast

9^{00} Conference opening

9^{05} Plenary talk: Charles Batty

10^{00} Coffee break

10^{30} Morning sessions

	T. Byczkowski and K. Bogdan		J. Voigt
10^{20}–10^{45}	Tomasz Byczkowski	10^{20}–10^{45}	Frank Neubrander
10^{50}–11^{15}	Tomasz Jakubowski	10^{50}–11^{15}	Sebastian Król
11^{20}–11^{45}	Jacek Dziubański	11^{20}–11^{45}	Alyona Zamyshlyaeva
11^{50}–12^{15}	Stanislav Stepin	11^{50}–12^{15}	Roland Schnaubelt
12^{20}–12^{45}	Alexander Bendikov	12^{20}–12^{45}	Josef Kreulich
12^{50}–13^{15}	Bartosz Trojan	12^{50}–13^{15}	Wolfgang Ruess

13^{15} Lunch

15^{00} Afternoon sessions (part 1):

	Y. Tomilov		A. Peris
15^{00}–15^{30}	Ralph Chill	15^{00}–15^{30}	José Bonet
15^{30}–16^{00}	David Seifert	15^{30}–16^{00}	Elisabetta Mangino
16^{00}–16^{30}	Tomasz Szarek	16^{00}–16^{30}	Alfred Peris

16^{30} Coffee break

17^{00} Afternoon sessions (part 2):

	Y. Tomilov		A. Peris
17^{00}–17^{30}	Piotr Rybka	17^{00}–17^{30}	Marcin Moszyński
17^{30}–18^{00}	Ernest Nieznaj	17^{30}–18^{00}	Félix Martínez-Giménez

18^{30} Dinner (barbecue)[1]

[1] If weather allows: otherwise barbecue will be arranged Tuesday

Tuesday

8^{00} Breakfast

9^{00} Plenary talk: Wolfgang Arendt

10^{00} Coffee break

10^{30} Morning sessions

	T. Byczkowski and K. Bogdan		J. Voigt
10^{20}–10^{55}	Jan Kisyński	10^{30}–10^{55}	András Bátkai
11^{00}–11^{25}	Agnieszka Kałamajska	10^{55}–11^{20}	Hendrik Vogt
11^{30}–11^{55}	Tomasz Grzywny	11^{20}–11^{45}	Chin Pin Wong
12^{00}–12^{25}	Victoria Knopova	11^{45}–12^{10}	Isabelle Chalendar
12^{30}–12^{55}	Bartłomiej Dyda	12^{10}–12^{35}	Bálint Farkas
12^{55}–13^{20}	Dominika Pilarczyk	12^{35}–13^{00}	Jürgen Voigt

13^{15} Lunch

15^{00} Afternoon sessions (part 1):

	Y. Tomilov		J. Banasiak
15^{00}–15^{30}	Charles Batty	15^{00}–15^{30}	Jerome Goldstein
15^{30}–16^{00}	Roland Schnaubelt	15^{30}–15^{50}	Valentina Parfenenkova
16^{00}–16^{30}	Markus Haase	15^{50}–16^{10}	Sophiya Zagrebina
		16^{10}–16^{30}	Henryk Leszczyński

16^{30} Coffee break

17^{00} Afternoon sessions (part 2):

	Y. Tomilov		J. Banasiak
17^{00}–17^{30}	Andrzej Palczewski	17^{00}–17^{20}	Rodrigue Yves M'pika Massoukou
17^{30}–18^{00}	Lassi Paunonen	17^{20}–17^{40}	Georgy Sviridyuk
		17^{40}–18^{00}	Jacek Banasiak

18^{15} Dinner

Wednesday

7^{30} Breakfast

8^{30} Plenary talk: Roberto Triggiani

9^{30} Coffee break

9^{50} Morning sessions

	I. Lasiecka, R. Triggiani, J. Zabczyk		A. Bobrowski
9^{50}–10^{15}	Jerzy Zabczyk	9^{50}–10^{15}	Markus Haase
10^{20}–10^{45}	George Avalos	10^{20}–10^{45}	Sebastian Król
10^{50}–11^{15}	Natalia Manakova	10^{50}–11^{15}	Delio Mugnolo
11^{20}–11^{45}	Irena Lasiecka	11^{20}–11^{45}	Adam Gregosiewicz

11^{55} Lunch

12^{30} Sightseeing

18^{30} Dinner

Thursday

8^{00} Breakfast

9^{00} Plenary talk: Krzysztof Bogdan

10^{00} Coffee break

10^{30} Morning sessions

	A. Rhandi		J. Janas
10^{30}–11^{00}	Giorgio Metafune	10^{30}–10^{55}	Vladimir Müller
11^{00}–11^{30}	Chiara Spina	10^{55}–11^{20}	Marek Ptak
11^{30}–12^{00}	Cristian Tacelli	11^{20}–11^{45}	Zbigniew Burdak
12^{00}–12^{30}	Natalia Ivanova	11^{45}–12^{10}	Artur Płaneta
12^{30}–13^{00}	Fatima Boudchich	12^{10}–12^{35}	Joanna Blicharz
		12^{35}–13^{00}	Elżbieta Król

13^{15} Lunch

15^{00} Afternoon sessions (part 1):

	A. Rhandi		J. Banasiak
15^{00}–15^{30}	Simona Fornaro	15^{00}–15^{30}	Mustapha Mokhtar-Kharroubi
15^{30}–16^{00}	Dominik Dier	15^{30}–15^{50}	Marcin Małogrosz
16^{00}–16^{30}	Marjeta Kramar Fijavž	15^{50}–16^{10}	Minzilia Sagadeeva
		16^{10}–16^{30}	Wilson Lamb

16^{30} Coffee break

17^{00} Afternoon sessions (part 2):

	A. Rhandi		J. Banasiak
17^{00}–17^{20}	Luca Lorenzi	17^{00}–17^{20}	Proscovia Namayanja
17^{20}–17^{40}	Luciana Angiuli	17^{20}–17^{40}	Jurij Kozicki
17^{40}–18^{00}	Waed Dada	17^{40}–18^{00}	Miroslaw Lachowicz

18^{30} Concert of Chamber Music

19^{30} Conference Dinner

Friday

8^{00} Breakfast

9^{00} Plenary talk: Jerome Goldstein

10^{00} Coffee break

10^{30} Morning sessions

	A. Rhandi		R. Rudnicki
10^{30}–11^{00}	Viktor Gerasimenko	10^{30}–11^{00}	Ryszard Rudnicki
11^{00}–11^{30}	Anna Karczewska	11^{00}–11^{30}	Przemysław Paździorek
11^{30}–12^{00}	Sergey Piskarev	11^{30}–12^{00}	Andrzej Tomski
12^{00}–12^{30}	Sami Mourou	12^{00}–12^{30}	Paweł Zwoleński
12^{30}–13^{00}	Abdelaziz Rhandi	12^{30}–13^{00}	Joanna Jaroszewska

13^{00} Conference closing

13^{15} Farewell lunch

14^{00} – 15^{00} Buses to Poznań.

Sessions

Plenary talks

1. Wolfgang Arendt, The Dirichlet-to Neumann operator by hidden compactness.
2. Charles Batty, Fine scales of decay of operator semigroups.
3. Krzysztof Bogdan, Perturbations of integral kernels.
4. Jerome Goldstein, Some biased remarks on the development of semigroups of operators.
5. Roberto Triggiani, Optimal polynomial decay via interplay between semigroup.

1. Approximation and perturbation of semigroups (J. Voigt)

1. András Bátkai, PDE approximation of large systems of differential equations.
2. Isabelle Chalendar, Lower estimates near the origin for functional calculus on operator semigroups.
3. Bálint Farkas, Operator splitting for delay equations.
4. Josef Kreulich, Asymptotic equivalence of evolution equations in Banach spaces.
5. Sebastian Król, Perturbations of generators of C_0-semigroups and resolvent decay.
6. Frank Neubrander, Laplace transform inversion and approximaton of semigroups.
7. Wolfgang Ruess, Invariant sets for semigroups of nonlinear operators.
8. Roland Schnaubelt, Splitting methods for Schrodinger equations with singular potentials.
9. Hendrik Vogt, A weak Gordon type condition for absence of eigenvalues of one-dimensional Schrödinger operators.
10. Jürgen Voigt, Perturbations for linear delay equations in L_p.
11. Chin Pin Wong, Honesty theory of positive perturbations.
12. Alyona A. Zamyshlyaeva, An alternative approximation of the degenerate strongly continuous operator semigroup.

2. Asymptotic behaviour of semigroups (J. Tomilov)

1. Charles Batty, Quasi-hyperbolic semigroups.
2. Ralph Chill, A Katznelson-Tzafriri theorem with rates for C_0-semigroups on Hilbert spaces.
3. Markus Haase, Convergence rates in the mean ergodic theorem for semigroups.
4. Ernest Nieznaj, Asymptotic behavior of a passive tracer in random fields.
5. Andrzej Palczewski, Convergence of semigroups associated to heat propagation models.
6. Lassi Paunonen, Robustness of polynomial stability of semigroups.
7. Piotr Rybka, A global attractor of a sixth order Cahn-Hilliard type equation.
8. Roland Schnaubelt, Strong convergence in L^p-spaces for invariant measures for non-autonomous Kolmogorov equations.
9. David Seifert, Rates of decay in the classical Katznelson-Tzafriri theorem.
10. Tomasz Szarek, Ergodic measures for Markov semigroups.

3. Cosine operator functions (A. Bobrowski)

1. Adam Gregosiewicz, Generation of moments-preserving cosine families by Laplace operators.
2. Markus Haase, Cosine functions and functional calculus.
3. Sebastian Król, Resolvent characterisation of generators of cosine functions and C_0-semigroups.
4. Delio Mugnolo, No boundary conditions for wave equations on an interval.

4. Heat kernels, Green's functions and Hardy spaces (B. Bogdan, T. Byczkowski)

1. Alexander Bendikov, On the spectrum of the hierarchical Laplacian.
2. Tomasz Byczkowski, Hitting half-spaces or spheres by Ornstein-Uhlenbeck type diffusions.

3. Bartłomiej Dyda, Sufficient and necessary conditions for fractional Hardy inequality.
4. Jacek Dziubański, On isomorphisms of Hardy spaces for certain Schrödinger operators.
5. Tomasz Grzywny, Heat kernel estimates for unimodal Levy processes.
6. Tomasz Jakubowski, Fundamental solution of fractional diffusion equation with singular drift.
7. Agnieszka Kałamajska, On solutions to heat equation with the initial condition in Orlicz-Slobodetskii space.
8. Jan Kisyński, Convolution operators as generators of one-parameter semigroups.
9. Victoria Knopova, On the parametrix solution to the Cauchy problem for some nonlocal operator.
10. Dominika Pilarczyk, Self–similar asymptotics of solutions to heat equation with inverse square potential.
11. Stanislav Stepin, Heat-type kernels: regularized traces and short-time asymptotics.
12. Bartosz Trojan, Heat kernel asymptotics on affine buildings.

5. Linear models in chaotic dynamics (A. Peris)

1. José Bonet, Mean ergodic semigroups on Frechet spaces.
2. Elisabetta Mangino, Spectral conditions for generators of distributional chaotic semigroups.
3. Félix Martínez-Giménez, The specification property for linear operators.
4. Marcin Moszyński, Discrete analogs of the asymptotic Levinson theorem and their spectral applications for Jacobi operators.
5. Alfred Peris, Strong mixing measures for C_0-semigroups.

6. Semigroups for evolution equations (A. Rhandi)

1. Luciana Angiuli, Hypercontractivity and long time behaviour in nonautonomous Kolmogorov equations.
2. Fatima Boudchich, Feedback stabilization of some functional differential equations.

3. Waed Dada, A semigroup approach to numerical ranges of operators.

4. Dominik Dier, Invariance of convex sets for non-autonomous evolution equations governed by forms.

5. Simona Fornaro, Semigroups generated by degenerate elliptic operators.

6. Viktor Gerasimenko, On the semigroups for quantum many-particle evolution equations.

7. Natalia Ivanova, Inverse problem for a degenerate evolution equation with overdetermination on the solution semigroup kernel.

8. Anna Karczewska, Resolvent operators corresponding to linear Volterra equations.

9. Marjeta Kramar Fijavž, The semigroup approach to dynamical processes in networks.

10. Luca Lorenzi, Heat kernel estimates for autonomous and nonautonomous evolution equations.

11. Giorgio Metafune, Weighted Rellich and Calderón-Zygmund inequalities in L^p.

12. Sami Mourou, Elliptic operators with complex unbounded coefficients on arbitrary domains L^p-theory and kernel estimatese.

13. Sergey Piskarev, The discretization of Bitzadze-Samarsky type inverse problemfor elliptic equations with Dirichlet and Neumann conditions.

14. Abdelaziz Rhandi, Kernel estimates for nonautonomous Kolmogorov equations.

15. Chiara Spina, Homogeneous Calderon-Zygmund estimates for a class of second order elliptic operators.

16. Cristian Tacelli, On Schrödinger operator with unbounded coefficients.

7. Semigroups in biology/Markov semigroups (R. Rudnicki)

1. Joanna Jaroszewska, Asymptotic properties of semigroups of Markov operators and of families of Markov-type nonlinear operators.

2. Przemysław Rafał Paździorek, Long time behaviour of the stochastic model of stem cells di erentiation with random switching.

3. Ryszard Rudnicki, Piece-wise deterministic processes in biological models.

4. Andrzej Tomski, The dynamics of enzyme inhibition controlled by piece-wise deterministic Markov proces.
5. Paweł Zwoleński, Phenotypic evolution of hermaphrodites.

8. Semigroups in natural sciences (J. Banasiak, W. Lamb)

1. Jacek Banasiak, Compactness and analyticity of fragmentation semigroups.
2. Jerome Goldstein, The deterministic PDEs of mathematical finance.
3. Jurij Kozicki, Markov evolution of a spatial logistic model: micro-and mesoscopic description.
4. Miroslaw A. Lachowicz, Semigroups in biology.
5. Wilson Lamb, Discrete coagulation-fragmentation equations.
6. Henryk Leszczyński, Semigroups and the maximum principle for structured populations with diffusion.
7. Marcin Małogrosz, Dimension reduction in a model of morphogen transport.
8. Rodrigue Yves M'pika Massoukou, Asymptotic analysis of a singularly perturbed nonlinear problem.
9. Mustapha Mokhtar-Kharroubi, Trend to equilibrium of conservative kinetic equations on the torus.
10. Proscovia Namayanja, Flow in networks with sinks.
11. Valentina Parfenenkova, Feynman-Kac theorem in Hilbert spaces.
12. Minzilia A. Sagadeeva, An evolution operator for the nonstationary Sobolev type equation.
13. Georgy A. Sviridyuk, Degenerate operator groups in the optimal measurement theory.
14. Sophiya A. Zagrebina, The degenerate operator groups theory and multipoint initial-finish problem for Sobolev type equations.

9. Semigroups of operators in control theory (I. Lasiecka, R. Triggiani, J. Zabczyk)

1. George Avalos, Concerning semigroups of fluid-structure PDE models.
2. Natalia A. Manakova, An optimal control over solutions of the initial-finish problem for one class of linear Sobolev type equations.
3. Irena Lasiecka, Global existence of solutions to a 3-D fluid structure interactions with moving interface.
4. Jerzy Zabczyk, Null controllable systems with vanishing energy.

10. Special classes of operators in Banach and Hilbert spaces (J. Janas)

1. Joanna Blicharz, Unitary N-dilations for tuples of commuting matrices.
2. Zbigniew Burdak, On the decomposition and the model for commuting isometries.
3. Elżbieta Król, Properties of generalized Toeplitz operators.
4. Vladimir Müller, On joint numerical radius.
5. Artur Płaneta, Automorphisms of multidimensional spectral order.
6. Marek Ptak, On the reflexivity, hyperreflexivity and transitivity of Toeplitz operators.

Hypercontractivity and long time behaviour in nonautonomous Kolmogorov equations

Luciana Angiuli
University of Salento, Italy

Semigroups for evolution equations

Joint work with Alessandra Lunardi and Luca Lorenzi.

We consider nonautonomous Cauchy problems,

$$\begin{cases} D_t u(t,x) = \mathcal{A}(t)u(t,x), & (t,x) \in (s,+\infty)\times\mathbb{R}^d, \\ u(s,x) = f(x), & x \in \mathbb{R}^d, \end{cases}$$

where $\{\mathcal{A}(t)\}_{t\in I}$ is a family of second order differential operators,

$$(\mathcal{A}(t)\zeta)(x) = \mathrm{Tr}(Q(t)D^2\zeta(x)) + \langle b(t,x), \nabla\zeta(x)\rangle,$$

with smooth enough coefficients $Q = [q_{ij}]_{i,j=1,\dots,d}$ and $b = (b_1,\dots,b_d)$, (possibly unbounded), defined in I and $I\times\mathbb{R}^d$, respectively, where I is an open right halfline and $s \in I$.
It is well known that the usual L^p spaces with respect to the Lebesgue measure dx are not a natural setting for elliptic and parabolic operators with unbounded coefficients, unless quite strong growth assumptions are imposed on their coefficients. Much better settings are L^p spaces with respect to the so called evolution systems of measures $\{\mu_t : t \in I\}$ associated to the evolution operator $G(t,s)$, i.e. a family of Borel probability measures in $\mathbb{R}^d$ satisfying

$$\int_{\mathbb{R}^d} G(t,s)f d\mu_t = \int_{\mathbb{R}^d} f d\mu_s =: m_s f, \qquad t > s \in I,\;\; f \in C_b(\mathbb{R}^d).$$

We prove hypercontractivity results in the spaces $L^p(\mathbb{R}^d,\mu_t)$ and we study the asymptotic behavior of $G(t,s)$ as $t \to +\infty$.
The starting point of our analysis is the proof of the logarithmic Sobolev inequality for the measures μ_t, in the form

$$\int_{\mathbb{R}^d} |f|^p \log|f|\, d\mu_t \le \frac{1}{p}\left(\int_{\mathbb{R}^d}|f|^p d\mu_t\right)\log\left(\int_{\mathbb{R}^d}|f|^p d\mu_t\right) + pC\int_{\mathbb{R}^d}|f|^{p-2}|\nabla f|^2\chi_{\{f\neq 0\}}d\mu_t, \quad (1)$$

for any $t \in I$, any $p \in (1,+\infty)$ and some positive constant C, independent of $f \in C_b^1(\mathbb{R}^d)$, t and p.

The logarithmic Sobolev inequality has a crucial role in the proof of the hypercontractivity results in the spaces $L^p(\mathbb{R}^d,\mu_t)$ which, together with the Poincaré inequality, allow us to compare the asymptotic behavior of $\|G(t,s)f - m_s f\|_{L^p(\mathbb{R}^d,\mu_t)}$ and $\|\,|\nabla_x G(t,s)f|\,\|_{L^p(\mathbb{R}^d,\mu_t)}$.

References

[1] L. Angiuli, (2013), *Pointwise gradient estimates for evolution operators associated with Kolmogorov operators*, Arch. Math. (Basel), **101**, 159-170.
[2] L. Angiuli, L. Lorenzi, A. Lunardi, (2013), *Hypercontractivity and asymptotic behaviour in nonautonomous Kolmogorov equations*, available at
arXiv:1203.1280v1. Comm. Partial Differential Equations (to appear).
[3] M. Kunze, L. Lorenzi, A. Lunardi, (2010), *Nonautonomous Kolmogorov parabolic equations with unbounded coefficients*, Trans. Amer. Math. Soc., **362**, 169-198.

The Dirichlet-to Neumann operator by hidden compactness

Wolfgang Arendt
University of Ulm, Germany

Plenary talk

The Dirichlet-to Neumann operator is a selfadjoint operator defined on the L^2 space of the boundary of a bounded open set. It can be defined most conveniently using form methods, and actually it is the prototype example for applying new arguments established together with ter Elst [1]. We will explain in more detail these form methods. They allow one to associate a DtN operator not only to the Laplacian but to an arbitrary elliptic operator. A delicate situation occurs if 0 is in the spectrum of the realization of this elliptic operator with Dirichlet boundary conditions. Then we use a new method which we call "hidden compactness". It is based on a version of the Lax-Milgram Lemma involving the Fredholm alternative. In this somehow singular case, the corresponding DtN operator is actually a self-adjoint graph (but its resolvent is still a single-valued operator). Still, this case is of particular importance and not just a generalization, and this for two reasons. If one wants to consider convergence of DtN-operators, for example if the coefficients vary, then one has to pass over the singular points. Surprisingly, the unique continuation property plays an important role to establish convergence theorems. The second reason concerns Friedlander's theorem on spectral inclusion of Dirichlet and Neumann eigenvalues. Here the singular case has to be considered if one wants to prove the strict inequality [3].

The talk is based on common work [2] with Tom ter Elst, James Kennedy and Manfred Sauter.

References

[1] W. Arendt, A.F.M. ter Elst: Sectorial forms and degenerate differential operators. J. Operator Th. 67 (2012). 33-72.
[2] W. Arendt, A.F.M. ter Elst, J. B. Kennedy, M. Sauter: The Dirichlet-to-Neumann operator via hidden compactness. ArXiv: 1305.0720. To appear in J. Funct. Anal.
[3] W. Arendt, R. Mazzeo: Friedlander's eigenvalue inequalities and the Dirichlet-to-Neumann semigroup. Commun. Pure Appl. Anal. 11 (2012), 2201-2212.

Compactness and analyticity of fragmentation semigroups

Jacek Banasiak **Semigroups in natural sciences**
University of KwaZulu-Natal & Technical University of Łódź

We consider discrete fragmentation models and present recent results on analyticity and compactness of the fragmentation semigroup. These results allow for proving a number of properties concerned with the long term behaviour of such semigroups, such as the asynchronous growth (decay) property and also some asymptotic properties. We also provide a number of counterexamples, showing that not all fragmentation semigroups are analytic and compact.

References

[1] J. Banasiak and W. Lamb, *The discrete fragmentation equation: semigroup, compactness and asynchronous exponential growth*, Kinetic and Related Models, 5(2), (2012), 223-236.
[2] J. Banasiak, *Transport processes with coagulation and strong fragmentation*, Discrete and Continuous Dynamical Systems - Series B 17 (2), (2012), 445-472.
[3] J. Banasiak, *Global classical solutions of coagulation-fragmentation equations with unbounded coagulation rates*, Nonlinear analysis: Real World applications, 13, (2012), 91-105.
[4] J. Banasiak, *On an irregular dynamics of certain fragmentation semigroups*, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas, 105, (2011), 61-377.

PDE approximation of large systems of differential equations

András Bátkai
Eötvös Loránd University, Hungary

Approximation and perturbation of semigroups

A large system of ordinary differential equations is approximated by a parabolic partial differential equation with dynamic boundary condition and a different one with Robin bondary condition. Using the theory of differential operators with Wentzell boundary conditions and similar theories, we give estimates on the order of approximation. The theory is demonstrated on a voter model where the Fourier method applied to the PDE seems to be of great advantage.

References

[1] Bátkai, A., Havasi, Á., Horváth, R., Kunszenti-Kovács, D., Simon, P. L., *PDE approximation of large systems of differential equations*, Preprint, 2013, `arXiv:1303.6235`

Fine scales of decay of operator semigroups

Charles Batty
University of Oxford, United Kingdom

Plenary talk

A very efficient way to obtain rates of energy decay for damped equations is to use operator semigroups to pass from resolvent estimates to energy estimates. This is known to give the optimal results in cases when the resolvent estimates have simple forms such as being exactly polynomial ($|s|^{\alpha}$). This talk will review that theory and also cases when the resolvent estimates are slightly different.

Quasi-hyperbolic semigroups

Charles Batty
University of Oxford, United Kingdom

Asymptotic behaviour of semigroups

This talk will describe a class of C_0-semigroups which are not necessarily hyperbolic but behave similarly. The failure of spectral mapping theorems prevents a simple characterisation of quasi-hyperbolicity in terms of the generator, so we discuss properties of semigroups which can be deduced from the appropriate conditions on the generator.

On the spectrum of the hierarchical Laplacian

Alexander Bendikov
Wrocław University, Poland

Heat kernels, Green's functions and Hardy spaces

Let (X, d) be a locally compact separable ultra-metric space. We assume that (X, d) is proper, that is, any closed ball $B \subset X$ is a compact set. Given a measure m on X and a function $C(B)$ defined on the set of balls (the choice function) we define the hierarchical Laplacian L_C which is closely related to the concept of the hierarchical lattice of F.J. Dyson, *Existence of a phase-transition in a one-dimensional Ising ferromagnet*, Comm. Math. Phys. **12** (1969).

L_C is a non-negative definite self-adjoint operator in $L^2(X, m)$. We address in our talk the following question: *How general can be the set* $Spec(L_C) \subseteq \mathbb{R}_+$?

When (X, d) is compact, $Spec(L_C)$ is an increasing sequence of eigenvalues of finite multiplicity which contains 0. Assuming that (X, d) is not compact we show that under some natural conditions concerning the structure of the hierarchical lattice ($\equiv$ the tree of d-balls) any given closed subset $M \subseteq \mathbb{R}_+$ which accumulates at 0 may appear as $Spec(L_C)$ for some appropriately chosen function $C(B)$. We apply our results to studying the operator of fractional derivative of V.S. Vladimirov, *Generalized functions over the field of p-adic numbers*, Uspekhi Mat. Nauk **43** (1988), and its random perturbations defined on the field of p-adic numbers.

This is joint work with Pawel Krupski (MI Wroclaw University).

Perturbations of integral kernels

Krzysztof Bogdan **Plenary talk**
Wrocław University of Technology, Poland

I will discuss joint work with Wolfhard Hansen, Tomasz Jakubowski, Sebastian Sydor and Karol Szczypkowski on Schrödinger-type perturbations of integral kernels on space-time. In the case of transition kernels and potential kernels, the perturbations generally correspond to adding an integral term to the generator. We give explicit estimates for the resulting kernels under a natural condition on the first nontrivial term in the perturbation series. The condition is flexible enough for kernels with power-type asymptotics, specifically if 3G Theorem holds for the kernel. We indicate modifications required to handle Gaussian kernels by means of a 4G Theorem. We also discuss non-local perturbations, which model evolution of mass in presence of dislocations.

References

[1] Bogdan, K., Hansen, W., Jakubowski, T. (2008) *Time-dependent Schrödinger perturbations of transition densities*, Studia Math. 189, no. 3, 235–254.
[2] Bogdan, K., Jakubowski, T., Sebastian, S. (2012) *Estimates of perturbation series for kernels*, J. Evol. Equ. 12, no. 4, 973–984.
[3] Bogdan, K., Hansen, W., Jakubowski, T. (2013) *Localization and Schrödinger perturbations of kernels*, Potential Analysis 39, no. 1, 13–28.
[4] Bogdan, K., Szczypkowski, K. (2013) *Gaussian estimates for Schrödinger perturbations*, arXiv1301.4627.
[5] Bogdan, K, Sydor, S. (2013) *On nonlocal perturbations of integral kernels*, arXiv:1205.4571.

Mean ergodic semigroups on Féchet spaces

José Bonet
Polytechnic University of Valencia, Spain

Linear models in chaotic dynamics

We report on joint work with Angela A. Albanese (Univ. Lecce, Italy) and Werner J. Ricker (Univ. Eichstaett, Germany).

We present criteria for determining (uniform) mean ergodicity of C_0-semigroups of linear operators in a sequentially complete, locally convex Hausdorff space X. A characterization of reflexivity (and of the property of being Montel) of complete, barrelled spaces X with a basis in terms of (uniform) mean ergodicity of certain C_0-semigroups acting in the space, is presented. Examples of C_0-semigroups on Köthe echelon spaces and on certain Fréchet function spaces is also included.

References

[1] Albanese, A.A., Bonet, J., Ricker, W.J. (2012), Mean ergodic semigroups of operators, RACSAM 106, 299-319.
[2] Albanese, A.A., Bonet, J., Ricker, W.J. (2013), Montel resolvents and uniformly mean ergodic semigroups of linear operators, Quaestiones Math. 36, 253-290.
[3] Albanese, A.A., Bonet, J., Ricker, W.J. (2013), Uniform mean ergodicity of C_0-semigroups in a class of Fréchet spaces, Functiones Approx. (to appear).

Feedback stabilization of some functional differential equations

Fatima Boudchich
Cadi Ayyad University of Marrakech, Morocco

Semigroups for evolution equations

Khalil Ezzinbi
Cadi Ayyad University of Marrakech, Morocco

In this work we study the stabilization for some partial functional differential equations on Banach spaces. We suppose that the linear part is not necessarily densely defined and satisfies the well known Hille-Yosida condition. Assuming that the semigroup of operators associated to the undelayed equation is compact, we characterize those systems that can be stabilized using a feedback control.
Keywords: Stabilization, C_0-semigroup, Infinite dimensional spaces, Retarded Functional Differential Equations.
Stability is an important aspect of systems theory. If a system is not stable we try to stabilize it as well as possible,this process is called stabilizability and stabilization. In many cases physical, biological or economical phenomena depend not only on the present state but also on some past occurrences, the importance of study of delay differential equations is well recognized in a wide range of applications, particularly the stabilisation using a feedback with past can be more interesting and efficient. Our purpose is to study the stabilization problem of the following partial functional differential equation:

$$\begin{cases} x'(t) = Ax(t) + L(x_t) + Bu(t) & t \geq 0, \\ x_0 = \varphi \in \mathcal{B}. \end{cases}$$

where $A : D(A) \to X$ is a Hille-Yosida operator, not necessarily densely defined on a Banach space X, $\mathcal{B}$ is a normed linear space of functions mapping $(-\infty, 0]$ to X and satisfying some fundamental axioms. $L : \mathcal{B} \to X$ is a bounded operator, $u(t) \in \mathbb{R}^m$ is the input in time t and $B : \mathbb{R}^m \to X$ is a linear map which represents the control action.

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Hitting half-spaces or spheres by Ornstein-Uhlenbeck type diffusions

Tomasz Byczkowski **Heat kernels, Green's functions and Hardy spaces**
Polish Academy of Sciences, Poland

In the talk we present a unified approach to compute harmonic measures of some domains $D \subset \mathbb{R}^n$ by various types of multidimensional diffusions. The basic diffusion under consideration is the Brownian motion with drift vector field F. We assume that F is potential, that is, it is the gradient of a scalar valued function V (called potential). We also assume that F is orthogonal to the boundary ∂D of the domain D. As an application we compute harmonic measures of half-spaces or balls for Laplace-Beltrami operator on hyperbolic spaces and for the classical Ornstein-Uhlenbeck operator. Methods of computation rely on stochastic calculus (Girsanov Theorem) as well as on the identification of some Brownian motion functionals and on the skew-product representation of multidimensional Brownian motion. We also extensively apply Laplace transformation method to obtain explicit representations of harmonic measures in terms of special functions (modified Bessel, Legendre, Whittaker and so on). In particular, for Ornstein-Uhlenbeck operator we obtain more complete result than the one published in [2]. The presentation is based on the paper [1].

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Lower estimates near the origin for functional calculus on operator semigroups

Isabelle Chalendar
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Approximation and perturbation of semigroups

We provide sharp lower estimates near the origin for the functional calculus $F(-uA)$ of a generator A of an operator semigroup defined either on the (strictly) positive real line or on a sector; here F is given either as the Laplace transform of a measure or distribution, or as the Fourier-Borel transform of an analytic functional. The results are linked to the existence of an identity element or an exhaustive sequence of idempotents in the Banach algebra generated by the semigroup. Both the quasinilpotent and non-quasinilpotent cases are considered, and sharp results are proved extending many in the literature.
This is joint work with Jean Esterle and Jonathan R. Partington (Bordeaux and Leeds)

A Katznelson-Tzafriri theorem with rates for C_0-semigroups on Hilbert spaces

Ralph Chill
Dresden University of Technology, Germany

Asymptotic behaviour of semigroups

The classical Katznelson-Tzafriri theorem, originally formulated for power bounded operators, states in one possible variant: if $(T(t))_{t\geq 0}$ is a bounded C_0-semigroup on a Banach space, with generator A, and if the spectrum of A on the imaginary axis contains at most the point 0, then $\lim_{t\to\infty} T(t)R(1,A) = 0$. More generally, if $f \in L^1(\mathbb{R}_+)$ is of spectral synthesis with respect to $\sigma(A) \cap i\mathbb{R}$, then $\lim_{t\to\infty} T(t)\hat{f}(T) = 0$. In this talk, we present a Katznelson-Tzafriri theorem for semigroups on Hilbert spaces which involves measures instead of L^1 functions and which gives, in a particular case, additional information about the decay rate to 0. This is joint work with Charles Batty and Yuri Tomilov.

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A semigroup approach to numerical ranges of operators

Waed Dada
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Semigroups for evolution equations

Based on the "Hille-Yosida theorem" and the "Lumer-Phillips Theorem", we define a numerical spectrum of a closed and densely defined operator on a Banach space. We discuss its properties and compare it to the numerical range.

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Invariance of convex sets for non-autonomous evolution equations governed by forms

Dominik Dier
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Semigroups for evolution equations

We consider a non-autonomous form $a : [0;T] \times V \times V \to C$ where V is a Hilbert space which is densely and continuously embedded in another Hilbert space H. Denote by $A(t) \in \mathfrak{L}(V, V')$ the associated operator. Given $f \in L^2(0, T, V')$, one knows that for each $u_0 \in H$ there is a unique solution $u \in H^1(0, T, V') \cap L^2(0, T, V)$ of

$$\dot{u}(t) + A(t)u(t) = f(t), u(0) = u_0.$$

This result by J. L. Lions is well-known. The aim of this talk is to present a criterion for the invariance of a closed convex subset $\mathcal{C}$ of H; i.e. we give a criterion on the form which implies that $u(t) \in \mathcal{C}$ for all $t \in [0;T]$ whenever $u_0 \in \mathcal{C}$. In the autonomous case for $f = 0$, the criterion is known and even equivalent to invariance by a result proved in [2]. We give applications to positivity and comparison of solutions to heat equations with non-autonomous Robin boundary conditions. This is a joint work with W. Arendt and E. M. Ouhabaz.

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Sufficient and necessary conditions for fractional Hardy inequality

Bartłomiej Dyda **Heat kernels, Green's functions and Hardy spaces**
Wrocław University of Technology, Poland

We will present sufficient conditions on a domain $D \subset \mathbb{R}^N$ and parameters s, p and β, so that the following (fractional) (s, p, β)-*Hardy inequality* hold

$$\int_D \frac{|u(x)|^p}{\delta_x^{sp}} \delta_x^\beta \, dx \leq c \int_D \int_D \frac{|u(x) - y(y)|^p}{|x-y|^{N+sp}} \delta_x^\beta \, dy \, dx \,, \quad u \in C_c(D). \tag{1}$$

Here $\delta_x = \mathrm{dist}(x, \mathbb{R}^N \setminus D)$.

We will also present a condition for capacity which is equivalent to (1).

The talk is based on joint preprints with Antti V. Vähäkangas [1, 2].

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On isomorphisms of Hardy spaces for certain Schrödinger operators

Jacek Dziubański
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Heat kernels, Green's functions and Hardy spaces

Jacek Zienkiewicz
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Let $\{K_t\}_{t>0}$ be the semigroup of linear operators on $\mathbb{R}^d$, $d \geq 3$, generated by a Schrödinger operator $L = \Delta - V$, where $V \geq 0$. We say that an L^1-function f belongs to the Hardy space H^1_L associated with L if the maximal function

$$\mathcal{M}f(x) = \sup_{t>0} |K_t f(x)|$$

belongs to $L^1(\mathbb{R}^d)$.

We shall prove that the following two conditions are equivalent:

(1) there is an L-harmonic function w, $0 < \delta \leq w(x) \leq C$, such that the mapping

$$H^1_L \ni f \mapsto wf \in H^1(\mathbb{R}^d)$$

is an isomorphism of the Hardy space H^1_L and the classical Hardy space $H^1(\mathbb{R}^d)$;

(2) the global Kato norm $\|V\|_{\mathcal{K}}$ is finite, where

$$\|V\|_{\mathcal{K}} = \sup_{x\in\mathbb{R}^d} \int_{\mathbb{R}^d} |x-y|^{2-d} V(y)\, dy.$$

The second result states that in this case the operator $(-\Delta)^{1/2} L^{-1/2}$ is an another isomorphism of the spaces H^1_L and $H^1(\mathbb{R}^d)$.

As corollaries we obtain that the space H^1_L admits:

(3) atomic decomposition with atoms satisfying the support condition $\operatorname{supp} a \subset B$ (for a certain ball B), the size condition $\|a\|_{L^\infty} \leq |B|^{-1}$, and the cancellation condition $\int a(x)w(x)dx = 0$

(4) characterization by the Riesz transforms $R_j = \partial_{x_j} L^{-1/2}$.

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Operator splitting for delay equations

Bálint Farkas **Approximation and perturbation of semigroups**
University of Wuppertal, Germany

In this talk we will consider delay equations of the form

$$\begin{aligned} \frac{\mathrm{d}u(t)}{\mathrm{d}t} &= Bu(t) + \Phi u_t, \qquad t \geq 0, \\ u(0) &= x \in E, \\ u_0 &= f \in L^p\big([-1,0];E\big). \end{aligned}$$

for the E-valued unknown function u, where E is a Banach space, B is the generator of a (linear) C_0-semigroup on E, u_t is the history function defined by $u_t(s) = u(t+s)$ and Φ is the delay operator. We will employ the semigroup approach on L^p-phase space (in the spirit of [4] and [5]) to be able to apply numerical splitting schemes to this problem. We prove convergence of theses schemes, investigate their convergence order in various situations: point or distributed delays, and even for nonlinear delay operators (based on [5]). We also intend to present some results for the nonautonomous case, and to present numerical examples as illustration. The talk is based on joint works with András Bátkai, Petra Csomós and Gregor Nickel.

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The semigroup approach to dynamical processes in networks

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Semigroups for evolution equations

We consider (nonautonomous) transport and diffusion equations

$$\dot{u}_j(t,x) = u_j'(t,x) \quad \text{and} \quad \dot{u}_j(t,x) = u_j''(t,x)$$

taking place on the edges of a finite connected network. At the vertices of the network we impose (nonautonomous) Kirchhoff-type conditions. We first rewrite these equations as a (nonautonomous) abstract Cauchy problem

$$\begin{cases} \dot{v}(t) = A(t)v(t), & t \geq 0, \\ v(0) = v_0 \in X, \end{cases}$$

on the appropriate Banach/Hilbert space X. The boundary conditions at the vertices of the network are contained in the domain of the operator $D(A(t)) \subset X$. We use semigroup and form methods to show wellposedness and study the long-term behavior of the solutions to the presented problems.

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Semigroups generated by degenerate elliptic operators

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Semigroups for evolution equations

The present talk addresses the following problem: given a second-order elliptic operator on a domain $\Omega \subset \mathbb{R}^n$

$$A = \sum_{i,j=1}^{n} a_{ij}\partial_{ij} + \sum_{i=1}^{n} b_i \partial_i,$$

whose diffusion coefficients vanish approaching the boundary, i.e.

$$\lim_{x \to \partial\Omega} a_{ij}(x) = 0 \quad \text{for some/all } i, j$$

does A generate an analytic semigroup in $L^p(\Omega)$ or $C(\overline{\Omega})$? Under which (if any) boundary conditions? Is it possible to characterize the domain in $L^p(\Omega)$? We will answer to the above questions in some special relevant cases, namely when the operator A belongs to one of the classes whose models on the halfspace $(x, y) \in \mathbb{R}^{n-1} \times (0, \infty)$ are given by

$$A_f = -y(\triangle_x + \partial_{yy}) + a \cdot \triangle_x + b\partial_y \quad \text{full degeneracy}$$
$$A_t = -y\triangle_x + \partial_{yy} + a \cdot \triangle_x + b\partial_y \quad \text{tangential degeneracy}$$

with $a \in \mathbb{R}^{n-1}$, $b \in \mathbb{R}$. The results have been obtained in collaboration with G. Metafune, D. Pallara, R. Schnaubelt and J. Prüss.

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On the semigroups for quantum many-particle evolution equations

Viktor Gerasimenko **Semigroups for evolution equations**
Institute of Mathematics of NAS of Ukraine, Ukraine

We review some recent results concerning theory of semigroups for quantum many-particle evolution equations.

The concept of cumulants (semi-invariants) of semigroups of operators forms the basis of the solution expansions for hierarchies of evolution equations of quantum many-particle systems, namely in case of the von Neumann hierarchy for correlation operators, the dual quantum BBGKY hierarchy for marginal observables, the quantum BBGKY hierarchy for marginal density operators and the nonlinear quantum BBGKY hierarchy for marginal correlation operators, as well as it underlies of the description of the kinetic evolution. For example, the nonperturbative solutions of the Cauchy problem of the dual quantum BBGKY hierarchy and the quantum BBGKY hierarchy are represented in the form of the expansions over particle clusters which generating operators are the corresponding-order cumulants of groups of operators of the Heisenberg equations and the von Neumann equations, respectively.

In particular, it is established that the cumulant structure of a solution of the von Neumann hierarchy for correlation operators induces the cumulant structure of solution expansions both the initial-value problem of the quantum BBGKY hierarchy for marginal density operators and the nonlinear quantum BBGKY hierarchy for marginal correlation operators. Thus, the dynamics of infinite-particle systems is governed by the dynamics of correlations.

Moreover, using the properties of cumulants of asymptotically perturbed groups of operators, the mean field asymptotic behavior of constructed solutions is established.

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Generation of moments-preserving cosine families by Laplace operators

Adam Bobrowski
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Cosine operator functions

Let L be the Laplace operator in $C[0,1]$ with domain $D(L) = C^2[0,1]$. Denote by $\mathfrak{L}_c$ the class of operators which are restrictions of L to various domains and generate strongly continuous cosine families in $C[0,1]$. Also, for non-negative integer k, let F_k be a linear functional in $C[0,1]$ given by

$$F_k f = \int_0^1 x^k f(x)\,\mathrm{d}x.$$

We say that the cosine family $\{C_A(t), t \in \mathbb{R}\}$ generated by $A \in \mathfrak{L}_c$ preserves the k-th moment about 0 iff

$$F_k C_A(t) f = F_k f, \qquad f \in C[0,1], t \in \mathbb{R}.$$

Let i and j be two non-negative integers such that $i < j$. We prove that there exists operator $A \in \mathfrak{L}_c$ such that the related cosine family preserves moments of order i and j about 0 if and only if $i = 0$. Moreover, if such operator exists it is unique. We will also discuss the case of non-integer i, j.

This result is a generalization of the theorem proved recently by A. Bobrowski and D. Mugnolo [1] in which the case $j = 1$ was considered.

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Heat kernel estimates for unimodal Lévy processes

Tomasz Grzywny
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Heat kernels, Green's functions and Hardy spaces

We present sharp bounds for transition densities $p_t(x)$, of isotropic unimodal Lévy processes on $\mathbb{R}^d$ (i.e. rotation invariant Lévy process with absolutely continuous Lévy measure which density is radially non-increasing), when their Lévy-Khintchine exponent ψ has weak local scaling at infinity of order strictly between 0 and 2. Our estimates may be summarized as follows,

$$p_t(x) \approx \left[\psi^{-1}\left(1/t\right)\right]^d \wedge \frac{t\psi^*(|x|^{-1})}{|x|^d},$$

where $\psi^*(r) = \sup_{|x|\le r} \psi(x)$. In fact, we show that the above estimate holds if and only if ψ has the weak local scaling. Moreover, this bounds is equivalent to bounds of the density of the Lévy measure.

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Convergence rates in the mean ergodic theorem for semigroups

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Asymptotic behaviour of semigroups

Given a strongly continuous and uniformly bounded semigroup $(T(t))_{t\geq 0}$ with generator $-A$ on a Banach space X, the Cesaro averages

$$C_t(A)x := \frac{1}{t}\int_0^t T(s)x\,\mathrm{d}s$$

converge to 0 as $t \to \infty$ if and only if $x \in \overline{\mathrm{ran}}(A)$. Apart from very special cases, there is no uniform rate in this convergence. However, such rates may well be observed on certain subspaces. In my talk I shall highlight how such subspaces can be conveniently described as $\mathrm{ran}\, g(A)$, where g is a *Bernstein function.* The associated convergence rate is easily read off from the function g. For so-called *special Bernstein functions* g these convergence rates are optimal under natural spectral conditions.

From this first step one obtains further sufficient criteria for convergence rates. For example, if μ is a positive Laplace transformable Radon measure on $[0,\infty)$ and $x \in X$ is such that

$$\lim_{\alpha\searrow 0}\int_0^\infty e^{-\alpha t}T(t)x\,\mu(\mathrm{d}t)$$

exists weakly, then $C_t(A)x = O(1/f(1/t))$ as $t \to \infty$, where f is the Laplace transform of μ.

The talk is based on joint work with A. Gomilko and Y. Tomilov [1,2].

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Cosine functions and functional calculus

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Cosine operator functions

In my talk I shall report on the functional calculus approach to cosine operator functions. Starting with an operator A with spectrum in a parabola and satisfying certain resolvent growth conditions one constructs a *holomorphic* functional calculus that allows to form the operator family $\mathrm{Cos}_A(t)$, $t \in \mathbb{R}$, as unbounded closed operators. A generation theorem emerges that is closely related to the complex inversion formula for the Laplace transform.

On the other hand, given that A is indeed the generator of a cosine function $(C(t))_{t\in\mathbb{R}}$ with associated sine function $(S(t))_{s\in\mathbb{R}}$, one can define a *Hille–Phillips* type functional calculus for A. The "decoupling identity"

$$C(s+t) = C(s)C(t) + AS(s)S(t) \qquad (s,t \in \mathbb{R})$$

is the key to a transference principle with interesting consequences.

The talk is based on [1]. The second part extends and simplifies results from [4] and [2] and is related to [3].

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Inverse problem for a degenerate evolution equation with overdetermination on the solution semigroup kernel

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Semigroups for evolution equations

The inverse problem for a linearized quasi-stationary phase field model is explored. This problem is reduced to a linear inverse problem for the first order differential equation in a Banach space with a degenerate operator at the derivative and an overdetermination condition on the solution semigroup kernel. The theorem on unique solvability for the inverse problem is obtained by virtue of the theory degenerate operator semigroups methods [1] as in [2] a nonlinear inverse problem for a hydrodynamical equations systems was researched.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a smooth boundary $\partial\Omega$, $T > 0$, β, $\delta \in \mathbb{R}$. Consider the initial-boundary value problem

$$(\beta + \Delta)(v(x,0) - v_0(x)) = 0, \quad x \in \Omega, \tag{1}$$

$$(1-\delta)v + \delta\frac{\partial v}{\partial n}(x,t) = (1-\delta)w + \delta\frac{\partial w}{\partial n}(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,T], \tag{2}$$

for the system of equations

$$v_t(x,t) = \Delta v(x,t) - \Delta w(x,t) + b_1(x,t)u(t), \quad (x,t) \in \Omega \times [0,T], \tag{3}$$

$$0 = v + (\beta + \Delta)w + b_2(x,t)u(t), \quad (x,t) \in \Omega \times [0,T], \tag{4}$$

with overdetermination condition on the subspace of degeneracy

$$\int\limits_{\Omega} K(y)w(y,t)dy = \psi(t), \quad (x,t) \in \Omega \times [0,T]. \tag{5}$$

Up to a linear change of functions $v(x,t)$, $w(x,t)$, the system coincides with the linearization of the quasistationary phase-field model [3], describing phase transitions of the first kind in terms of the mesoscopic theory. The unknown functions of the inverse problem (1)–(5) are $v(x,t)$, $w(x,t)$, $u(t)$.

Denote $Aw = \Delta w$, $D_A = H^2_\delta(\Omega) \subset L_2(\Omega)$, $\langle\cdot,\cdot\rangle$ is inner product in $L_2(\Omega)$. Let $\{\varphi_k : k \in \mathbb{N}\}$ be orthonormal in $L_2(\Omega)$ eigenfunctions of the operator A, enumerated with respect to the nonascending order of the eigenvalues $\{\lambda_k : k \in \mathbb{N}\}$, counting their multiplicities.

Theorem 1. *Let $-\beta \in \sigma(A)$, $b_i \in C^1([0,T]; L_2(\Omega))$, $i = 1, 2$, and $\langle b_1(\cdot,t), \varphi_k\rangle = 0$ for $\lambda_k \neq -\beta$, $K \in L_2(\Omega)$, $\langle K, \varphi_k\rangle = 0$ for $\lambda_k = -\beta$, $\langle K, b_2(\cdot,t)\rangle \neq 0$ for all $t \in [0,T]$, $\psi \in C^1[0,T]$, $v_0 \in H^2_\delta(\Omega)$. Then there exists a unique solution of the problem (1)–(5).*

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Fundamental solution of fractional diffusion equation with singular drift

Tomasz Jakubowski **Heat kernels, Green's functions and Hardy spaces**
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I will discuss a joint work with Grzegorz Karch and Jacek Zienkiewicz. We consider the fractional Laplacian $\Delta^{\alpha/2}$, where $\alpha \geq 1$, with divergence free drift satisfying estimates $|b(x)| \leq C|x|^{1-\alpha}$. We show that the fundamental solution $P(t,x,y)$ of this operator has global in time estimates $P(t,x,y) \leq ct^{-d/\alpha} \wedge t|x-y|^{-d-\alpha}$.

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Asymptotic properties of semigroups of Markov operators and of families of Markov-type nonlinear operators

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Semigroups in biology/Markov semigroups

I will present my recent work on the semigroups of Markov operators and on the families of Markov-type nonlinear operators acting on measures. I will start with the discussion of the relationships between various asymptotic properties of Markov semigroups such as the asymptotic strong Feller property, the e-property and the asymptotic e-property. Next I will present the criteria for the existence of invariant probability measures and their asymptotic stability, valid for Fellerian as well as non-Fellerian semigroups. Finally I will discuss variants of these results valid for general families of Markov-type nonlinear operators. I will also show some applications to iterated function systems.

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On solutions to heat equation with the initial condition in Orlicz-Slobodetskii space

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Heat kernels, Green's functions and Hardy spaces

We study the boundary-value problem

$$\tilde{u}_t = \Delta_x \tilde{u}(x,t), \quad \tilde{u}(x,0) = u(x),$$

where $x \in \Omega, t \in (0,T)$, $\Omega \subseteq \mathbf{R}^n$ is a bounded Lipschitz boundary domain, u belongs to certain Orlicz-Slobodetskii space $Y^{R,R}(\Omega)$. Under certain assumptions on the Orlicz function R, we prove that the solution u belongs to Orlicz-Sobolev space $W^{1,R}(\Omega \times (0,T))$. Links with trace embedding theorem from Sobolev space $W^{1,R}(\tilde{\Omega})$ defined on domain $\tilde{\Omega}$ into Orlicz-Slobodetski type space defined on the boundary of the domain $\partial\tilde{\Omega}$, will also be discussed. The talk will be based on results [1], [2], [3] and [4].

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Resolvent operators corresponding to linear Volterra equations

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Semigroups for evolution equations

The talk will be devoted to resolvent operators appearing during the study of Volterra equations of the form

$$u(t) = f(t) + \int_0^t [a(t-s) + (a*k)(t-s)]\, Au(s)\, ds + \int_0^t b(t-s)\, u(s)\, ds, \qquad (1)$$

$t \in [0,T]$, $T < \infty$. The operator A is a closed linear unbounded operator in Banach space B with a dense domain $D(A)$ equipped with the graph norm. We assume that $a, k, b \in L^1_{\mathrm{loc}}(\mathbb{R}_+;\mathbb{R})$ and that f is a continuous B-valued function.

The resolvent approach to the equations (1) is a generalization of the semigroup approach usually used with differential equations.

The resolvent operators considered, denoted by $\mathcal{R}(t)$, $t \geq 0$, are generated by the operator A and the kernel functions a, b, k.

In the presentation we provide the existence and convergence results of the resolvent operators considered. The results discussed play an important role in the study of stochastic versions of the Volterra equations (1). The presentation is based on joint papers with Carlos Lizama.

Convolution operators as generators of one-parameter semigroups

Jan Kisyński

Theorem 1. *Let* $G \in \widetilde{O}'_C(\mathbb{R}^n; M_{m\times m})$*, and let* E *be whichever of the following l.c.v.s.:* $S(\mathbb{R}^n;\mathbb{C}^m)$ *,* $D_{L^2}(\mathbb{R}^n;\mathbb{C}^m)$*,* $(\widetilde{O}_\mu)(\mathbb{R}^n;\mathbb{C}^m)$ *where* $\mu \in [0,\infty[$*, or* $S'(\mathbb{R}^n;\mathbb{C}^m) = S'(\mathbb{R}^n) \times \cdots \times S'(\mathbb{R}^n)$ *where each of the* m *factors is equipped with strong dual topology. Then* $(G*)_{|E} \in L(E;E)$ *and the following conditions are equivalent:*

(a) the weak Petrovskiĭ condition (independent of E*):* $0 \vee \max Re\ \sigma(\widehat{G}(\xi)) = O(\log|\xi|)$ *as* $\xi \in \mathbb{R}^n$ *and* $|\xi| \to \infty$*,*

(b) $(G*)_{|E}$ *is equal to the infinitesimal generator of a one-parameter semigroup* $(T_t)_{t\geq 0} \subset L(E;E)$ *of class* (C_0)*.*

The implication (a) $\Rightarrow$ (b) holds for a family of l.c.v.s. E continuously imbedded in $S'(\mathbb{R}^n;\mathbb{C}^m)$ larger than the family in Theorem 1. The proof of implication (b) $\Rightarrow$ (a) uses analytical tricks that depend on E.

Example. Let $m = n = 1$, $G = -\delta''$, and let E be whichever of the l.c.v.s. occurring in Theorem 1. Then $(G*)_{|E} \in L(E;E)$ and $(G*)_{|E}$ does not generate a semigroup $(T_t)_{t\geq 0} \subset L(E;E)$ of class (C_0). Indeed, $\widehat{G}(\xi) = \mathcal{F}(-\delta'')(\xi) = \xi^2$, therefore (a) is not satisfied.

On the parametrix solution to the Cauchy problem for some non-local operators

Victoria Knopova **Heat kernels, Green's functions and Hardy spaces**
V.M.Glushkov Institute of Cybernetics NAS of Ukraine, Ukraine

Consider the integro-differential equation

$$\frac{\partial}{\partial t}u(t,x) = L(x,D)u(t,x), \quad t>0, \quad x\in\mathbb{R}^n, \tag{1}$$

where the operator $L(x,D)$ is defined on functions ϕ from the Schwartz space as

$$L(x,D)\phi(x) := a(x)\nabla\phi(x) + \int_{\mathbb{R}^n}\big(\phi(x+u)-\phi(x)-u\nabla\phi(x)1_{\{\|u\|\le 1\}}\big)\mu(x,du), \tag{2}$$

and the kernel $\mu(x,du)$ satisfies $\sup_x\int_{\mathbb{R}^n}(1\wedge\|u\|^2)\mu(x,du)<\infty$. By developing a version of the parametrix method, we prove the existence of the fundamental solution to (1), and construct the upper and lower estimates on this solution. We also show some applications of the obtained estimates.

The talk is based on the joint work with Aleksei Kulik.

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Markov evolution of a spatial logistic model: micro- and mesoscopic description

Jurij Kozicki
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Semigroups in natural sciences

Markov evolution of a continuum spatial logistic model is studied at micro-and mesoscopic levels. The model describes an infinite system of point particles in R^d, which reproduce themselves at distant points (dispersal) and die, independently and under the influence of each other (competition). The microscopic description is based on an infinite chain of linear equations for moment (correlation) functions, similar to the BBGKY hierarchy used in the Hamiltonian dynamics of continuum particle systems. The mesoscopic description is based on a nonlinear and nonlocal kinetic equation for the particle's density obtained from the mentioned chain via a scaling procedure. The main conclusion of the microscopic description is that the competition can prevent the system from clustering. A possible homogenization of the solutions to the kinetic equation in the long-time limit is also demonstrated.

Asymptotic equivalence of evolution equations in Banach spaces

Josef Kreulich
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Approximation and perturbation of semigroups

It is shown how the approach of Yosida approximation of the derivative serves to obtain new results for evolution systems. i.e.

$$u'(t) \in A(t)u(t) + \omega u(t) + f(t), u(0) = u_0, t \in \mathbb{R}^+, \tag{1}$$

and the corresponding approximative equation

$$\left(\frac{d}{dt}\right)_\lambda u_\lambda(t) \in A(t)u_\lambda(t) + \omega u_\lambda(t) + f(t), u(0) = u_0, t \in \mathbb{R}^+, \tag{2}$$

Criteria are given for the asymptotic equivalence of two different evolution systems, i.e.

$$\lim_{t\to\infty} \|U_A(t,s)x - U_B(t,s)x\| = 0,$$

where the evolution systems are generated by two different families of nonlinear and multivalued time dependent operators $A(t)$, and $B(t)$.

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Resolvent characterisation of generators of cosine functions and C_0-groups

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Approximation and perturbation of semigroups

We prove new characterisations of the cosine function generators and group generators on UMD spaces and discuss their application to some classical problems in the cosine function theory.

More precisely, we show that the above classes of operators can be characterised on UMD spaces by means of a complex inversion formula. This, in particular, allows us to provide a strikingly elementary proof of Fattorini's result on square root reduction for cosine function generators on UMD spaces.

Moreover, we prove a cosine function analogue of the Gomilko-Feng-Shi characterisation of semigroup generators and apply it to answer in affirmative a question of Fattorini on the growth bounds of perturbed cosine functions on Hilbert spaces.

We also discuss characterisations of the cosine function generators on Hilbert spaces which correspond to the well-known results on the boundedness of the H^∞ functional calculus for sectorial operators, such as the McIntosh characterisation in terms of square function estimates and the Fröhlich-Weis characterisation by means of dilation properties.

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Semigroups in biology

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Semigroups in natural sciences

The general approach that allows to construct the Markov processes describing various processes in mathematical biology (or in other applied sciences) is presented. The Markov processes are of a jump type and the starting point is the related linear equations. They describe at the micro–scale level the behavior of a large number N of interacting individuals (entities). The large individual limit ("$N \to \infty$") is studied and the intermediate level (the meso–scale level) is given in terms of nonlinear kinetic–type equations. Finally the corresponding systems of nonlinear ODEs (or PDEs) at the macroscopic level (in terms of densities of the interacting subpopulations) are obtained. Mathematical relationships between these three possible descriptions are presented and explicit error estimates are given. The general framework is applied to propose the microscopic and mesoscopic models that correspond to well known systems of nonlinear equations in biomathematics.

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Discrete coagulation-fragmentation equations

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Semigroups in natural sciences

Coagulation and fragmentation processes arise in a number of areas of pure and applied science. Examples include colloidal aggregation, blood clotting and polymer science. The usual starting point when developing a mathematical model of such processes is to regard the system under consideration as one consisting of a large number of clusters that can coagulate to form larger clusters or fragment into a number of smaller clusters. Under the assumption that each cluster of size n consists of n identical fundamental units (monomers), we obtain a discrete model of coagulation-fragmentation which takes the form of an infinite system of ordinary differential equations.

In this talk, the associated initial-value problem for this infinite-dimensional system will be expressed as a semi-linear abstract Cauchy problem, posed in a physically relevant Banach space. Perturbation results from the theory of semigroups of operators will be used to establish the existence and uniqueness of globally-defined, strongly differentiable, non-negative solutions for uniformly bounded coagulation rates but with minimal restrictions placed on the fragmentation rates.

In one specific case of a pure fragmentation process, in which no coagulation occurs, an interesting phenomenon arises due to the existence of an explicit solution, which despite satisfying homogeneous initial conditions in a pointwise manner, appears to emanate from an initial state that has unit mass. This apparent paradox will be explained in a satisfactory manner by using the theory of Sobolev towers.

A couple of recent extensions of the existence/uniqueness results discussed in the first part of the talk will also be mentioned briefly. The first is concerned with a system of clusters which are distinguished, not only by size, but also by shape. The second, due to Jacek Banasiak, employs theory associated with analytic semigroups to relax the assumption that the coagulation rates are uniformly bounded.

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Global existence of solutions to a 3-D fluid structure interactions with moving interface

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Semigroups of operators in control theory

Equations of fluid structure interactions are described by Navier Stokes equations coupled to a dynamic system of elasticity. The coupling is on a free boundary interface between the two regions. The interface is moving with the velocity of the flow. The resulting model is a quasilinear system with parabolic-hyperbolic coupling acting on a moving boundary. One of the main features and difficulty in handling the problem is a mismatch of regularity between parabolic and hyperbolic dynamics. The existence and uniqueness of smooth local solutions has been established by D. Coutand and S. Shkoller *Arch. Rational Mechanics and Analysis* in 2005. Other local wellposedness results with a decreased amount of necessary smoothness have been proved in a series of papers by I. Kukavica, A. Tuffaha and M. Ziane. The main contribution of the present paper is *global* existence of smooth solutions. This is accomplished by exploiting a natural damping occurring at the interface along with a propagation of maximal parabolic regularity enjoyed by one component of the system.

This work is joint with M. Ignatova (Stanford University), I. Kukavica (University of Southern California, Los Angeles) and A. Tuffaha (The Petroleum Institute, Abu Dhabi, UAE).

Semigroups and the maximum principle for structured populations with diffusion

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Semigroups in natural sciences

We study a size-structured model which describes the dynamics of one population with growth, diffusion, reproduction and mortality rates, i.e.

$$\begin{aligned} u_t(t,s) &= \left(d(s)u_s(t,s)\right)_s - \left(\gamma(s)u(t,s)\right)_s - \mu(s)u(t,s) \\ &\quad + \int_0^m \beta(s,y)u(t,y)\,dy + g(t,s), \quad s \in (0,m) \end{aligned}$$

with linear Feller boundary conditions

$$\begin{aligned} \left[\left(d(s)u_s(t,s)\right)_s\right]_{s=0} - b_0 u_s(t,0) + c_0 u(t,0) = 0 \\ \left[\left(d(s)u_s(t,s)\right)_s\right]_{s=m} + b_m u_s(t,m) + c_m u(t,m) = 0 \end{aligned}$$

and the initial condition

$$u(0,s) = \omega(s), \quad \omega(s) \geq 0.$$

The present paper raises and develops the ideas found in [1], where the autors showed that the size structured model with certain boundary conditions is governed by a positive quasicontractive semigroup on a biologically relevant state space. The advantage of the semigroup approach is that it enables the description of population processes as dynamical systems in the state space. It seems that positivity of solutions is technical and tedious in their semigroup setting, whereas our approach is straightforward. The asymptotic behaviour of solutions is deduced in our study simply by means of the maximum principle.

The aim of this article is to provide more precise attempts to asymptotic analysis in a Hilbert space where one can recognize a finite dimensional subspace attracting some solutions. We prove a weak maximum principle for structured populations models with dynamic boundary conditions. We establish existence and positivity of solutions of these models and investigate the asymptotic behaviour of solutions. In particular, we analyse so called *size profile*.

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An optimal control over solutions of the initial-finish problem for one class of linear Sobolev type equations

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Semigroups of operators in control theory

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A lot of initial-boundary value problems for the equations and the systems of equations not resolved with respect to time derivative are considered in the framework of abstract Sobolev type equations that make up the vast field of non-classical equations of mathematical physics. Let $\mathfrak{X}, \mathfrak{Y}$ and $\mathfrak{U}$ be the Hilbert spaces. The operators $L \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$, $M \in \mathcal{C}l(\mathfrak{X}; \mathfrak{Y})$ and (L,p)-sectorial [1], $p \in \{0\} \cup \mathbb{N}$ and $B \in \mathcal{L}(\mathfrak{U}; \mathfrak{Y})$. Consider the equation

$$L\dot{x} = Mx + y + Bu, \quad \ker L \neq \{0\}. \tag{1}$$

Here functions $y : (0,\tau) \subset \mathbb{R}_+ \to \mathfrak{Y}$, $u : (0,\tau) \subset \mathbb{R}_+ \to \mathfrak{U}\,(\tau < \infty)$. The theory of degenerate semigroups of operators [1] is a suitable mathematical tool for the study of such problems. We consider the initial-finish problem [2], that is, Sobolev type linear equation (1) with the conditions

$$P_{in}(x(0) - x_0) = 0,\ P_{fin}(x(\tau) - x_\tau) = 0. \tag{2}$$

Here $\tau \in \mathbb{R}_+$, x_0, $x_\tau \in \mathfrak{X}$, the operators P_{in}, P_{fin} are the relatively spectral projections acting in the space $\mathfrak{X}$. The initial-finish problem (1), (2) is a natural generalization of the Showalter –Sidorov problem, which is a generalization of the Cauchy problem. The conditions (2) are different from those previously studied in that one projection of the solution is given at the initial moment, and the other is given at the final moment of the considered time period. We are interested in optimal control problem, which is to find such a pair $(\hat{x}, \hat{u}) \in \mathfrak{X} \times \mathfrak{U}_{ad}$, for which the relation

$$J(\hat{x}, \hat{u}) = \inf_{(x,u) \in \mathfrak{X} \times \mathfrak{U}_{ad}} J(x,u), \tag{3}$$

wherein all pairs (x,u) satisfy the problem (1), (2), takes place. Here

$$J(x,u) = \mu \sum_{q=0}^{1} \int_0^\tau \|z^{(q)} - z_0^{(q)}\|_{\mathfrak{Z}}^2 dt + \nu \sum_{q=0}^{k} \int_0^\tau \left\langle N_q u^{(q)}, u^{(q)} \right\rangle_{\mathfrak{U}} dt$$

is a specially constructed cost functional, $u \in \mathfrak{U}_{ad}$ is the control, $\mathfrak{U}_{ad}$ is a closed and convex set in the control space $\mathfrak{U}$. The operators $N_q \in \mathcal{L}(\mathfrak{U})$, $q = 0,\, 1,\, \ldots,\, p+1$ are self-adjoint and positive definite; $z_0 = z_0(t)$ is the desired observation and $\mu, \nu \geq 0$, $\mu + \nu = 1$, $0 \leq k \leq p+1$. Consider the Hilbert space of observations $\mathfrak{Z}$ and the operator $C \in \mathcal{L}(\mathfrak{X}; \mathfrak{Z})$ defining the observation $z(t) = Cx(t)$. Note that if $x \in H^1(X)$, then $z \in H^1(Z)$.

Introduce the following conditions.

The L-spectrum of the operator M be represented in the form

$$\sigma^L(M) = \sigma^L_{fin}(M) \cup \sigma^L_{in}(M), \qquad (A1)$$

where $\sigma^L_{fin}(M)$ is contained in a bounded domain $\Omega \subset \mathbb{C}$ with a piecewise smooth boundary γ, and $\gamma \cap \sigma^L(M) = \emptyset$;

$$\mathfrak{X}^0 \oplus \mathfrak{X}^1 = \mathfrak{X} \ (\mathfrak{Y}^0 \oplus \mathfrak{Y}^1 = \mathfrak{Y}); \qquad (A2)$$

$$\text{the operator } L_1^{-1} \in \mathcal{L}(\mathfrak{Y}^1; \mathfrak{X}^1) \text{ exists.} \qquad (A3)$$

Construct the spaces

$$H^{p+1}(\mathfrak{Y}) = \{v \in L_2(0, \tau; \mathfrak{Y}) : v^{(p+1)} \in L_2(0, \tau; \mathfrak{Y}), p \in \{0\} \cup \mathbb{N}\}.$$

The space $H^{p+1}(\mathfrak{Y})$ is Hilbert, because we deal with the Hilbert space $\mathfrak{Y}$ endowed with the inner product

$$[v, w] = \sum_{q=0}^{p+1} \int_0^\tau \left\langle v^{(q)}, w^{(q)} \right\rangle_{\mathfrak{Y}} dt.$$

Theorem 1. [3] Let the operator M be (L, p)-sectorial, $p \in \{0\} \cup \mathbb{N}$ and conditions (A1)–(A3) are fulfilled. Then, for all $y \in H^{p+1}(\mathfrak{Y})$, $x_0, x_\tau \in \mathfrak{X}$ there exists a unique optimal control over solutions of the problem (1), (2).

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Spectral conditions for generators of distributional chaotic semigroups

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Linear models in chaotic dynamics

We report on a joint work with A.Albanese (Univ. del Salento, Italy), X. Barrachina and A. Peris (Univ. Politécnica Valencia, Spain).

In the last years the chaotic behaviour of orbits of strongly continuous one parameter semigroups has been investigated by various authors. Chaotic and hypercyclic semigroups were studied in a systematic way for the first time by Desch, Schappacher, and Webb (1997), who gave also a sufficient condition for chaoticity of a semigroups based on the analysis of the point spectrum of the generator of the semigroup. Since then, it has been shown that chaos appears in C_0-semigroups associated to "birth and death"equations for cell populations, transport equations, first order partial differential equations and diffusion operators as the Ornstein-Uhlenbeck operators.

Recently another notion of chaos has been studied in the infinite-dimensional linear setting, namely distributional chaos. This concept was introduced by Schweizer and Smítal for interval maps with the aim of unifying various notions of chaos and it strengths the Li-Yorke chaos.

Various results about distributional chaotic semigroups are presented, focusing on sufficient conditions based on the analysis of the spectrum of the generator.

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The specification property for linear operators

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Linear models in chaotic dynamics

We introduce the notion of the Specification Property (SP) for operators on Banach spaces, inspired by the usual one of Bowen for continuous maps on compact spaces. This is a very strong dynamical property related to the chaotic behaviour. Several general properties of operators with the SP are established. For instance, every operator with the SP is mixing, Devaney chaotic, and frequently hypercyclic. In the context of weighted backward shifts, the SP is equivalent to Devaney chaos. In contrast, there are Devaney chaotic operators (respectively, mixing and frequently hypercyclic operators) which do not have the SP. This is a joint work with S. Bartoll and A. Peris.

Weighted Calderón-Zygmund and Rellich inequalities in L^p

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Semigroups for evolution equations

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In 1956, Rellich proved the inequalities

$$\left(\frac{N(N-4)}{4}\right)^2 \int_{\mathbb{R}^N} |x|^{-4}|u|^2 dx \leq \int_{\mathbb{R}^N} |\triangle u|^2 dx$$

for $N \neq 2$ and for every $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$. These inequalities have been then extended to L^p-norms: in 1996, Okazawa proved the validity of

$$\left(\frac{N}{p}-2\right)^p \left(\frac{N}{p'}\right)^p \int_{\mathbb{R}^N} |x|^{-2p}|u|^p dx \leq \int_{\mathbb{R}^N} |\triangle u|^p dx$$

for $1 < p < \dfrac{N}{2}$. Weighted Rellich inequalities have also been studied. In 1998, Davies and Hinz obtained for $N \geq 3$ and for $2 - \dfrac{N}{p} < \alpha < 2 - \dfrac{2}{p}$

$$C(N,p,\alpha) \int_{\mathbb{R}^N} |x|^{(\alpha-2)p}|u|^p dx \leq \int_{\mathbb{R}^N} |x|^{\alpha p}|\triangle u|^p dx \tag{1}$$

with the optimal constants $C(N,p,\alpha) = \left(\dfrac{N}{p} - 2 + \alpha\right)^p \left(\dfrac{N}{p'} - \alpha\right)^p$. Later Mitidieri showed that (1) holds in the wider range $2 - \dfrac{N}{p} < \alpha < N - \dfrac{N}{p}$ and with the same constants. In a recent paper, in 2012, Caldiroli and Musina improved weighted Rellich inequalities for $p = 2$ by giving necessary and suffcient conditions on α for the validity of (1) and finding also the optimal constants $C(N,2,\alpha)$. In particular they proved that (1) is verified for $p = 2$ if and only if $\alpha \neq N/2 + n$, $\alpha \neq N/2 + 2 - n$ for every $n \in N_0$. Similar results have been also obtained by Ghoussoub and Moradifam under the restriction $\alpha \geq (4-N)/2$ and with different methods.

We extend Caldiroli-Musina result to $1 \leq p \leq \infty$, computing also best constants in some cases. We show that (1) holds if and only if $\alpha \neq N/p' + n$, $\alpha \neq -N/p + 2 - n$ for every

$n \in N_0$. Moreover, we use Rellich inequalities to find necessary and sufficient conditions for the validity of weighted Calderón-Zygmund estimates when $1 < p < \infty$

$$\int_{\mathbb{R}^N} |x|^{\alpha p} |D^2 u|^p dx \leq C \int_{\mathbb{R}^N} |x|^{\alpha p} |\triangle u|^p dx \tag{2}$$

for $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$. We find that (2) holds if and only if $\alpha \neq N/p' + n$ for every $n \in \mathbb{N}_0$ and, $\alpha \neq N/p + 2 - n$ for every $n \in \mathbb{N}$, $n \geq 2$.

Trend to equilibrium of conservative kinetic equations on the torus

Mustapha Mokhtar-Kharroubi
University of Franche-Comté, France

Semigroups in natural sciences

This work deals with relaxation phenomena to equilibrium for a general class of conservative neutron transport equations on the torus. We give a general compactness result in L^1 space and characterize the existence of a spectral gap for the corresponding semigroup. In absence of a spectral gap, we show also a strong convergence to equilibrium state relying on ergodic properties and (0-2) law for perturbed semigroups with "asymptotic smoothing effects".

Discrete analogs of the asymptotic Levinson theorem and their spectral applications for Jacobi operators

Marcin Moszyński **Special classes of operators in Banach and Hilbert spaces**
University of Warsaw, Poland

Finding asymptotic information on solutions x of the discrete system

$$x(n+1) = A(n)x(n), \qquad n \geq n_0, \tag{1}$$

where $x = \{x(n)\}_{n \geq n_0}$ is a sequence of $\mathbb{C}^d$ vectors, and $A = \{A(n)\}_{n \geq n_0}$ is a fixed sequence of $d \times d$ complex matrices, is a typical asymptotic problem for linear difference equations. The most "classical" result was probably the famous asymptotic Poincaré theorem, later improved by Perron. It was formulated for k-th order scalar difference equation but those results possess also generalizations [7, 8] for discrete systems of the above form (1).

The other group of results can be called "discrete Levinson type theorems" (**DLT**)and it contains discrete analogs of the classical Levinson theorem on the asymptotic behavior of solutions of ordinary differential equation

$$\frac{dy(t)}{dt} = \mathcal{A}(t)y(t), \qquad t \geq t_0,$$

where $\mathcal{A}(t)$ — a complex $d \times d$ matrix, $y(t)$ — a $\mathbb{C}^d$ vector.

One of the first discrete versions was published (without proof and also without some important assumptions) by Evgrafov in [4]. The main correct result in this area belongs to Benzaid and Lutz [1], where the so-called dichotomy conditions on A were formulated.

This talk is devoted to some old versions (e.g. [5, 6]) and also to the new version [9] of discrete Levinson theorem — for systems with so-called singular limit. All those versions concern various special assumptions on the matrix sequence A.

Several examples of applications of **DLT** to spectral studies of Jacobi Operators will be shown.

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Elliptic operators with complex unbounded coefficients on arbitrary domains L^p-theory and kernel estimatese

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Semigroups for evolution equations

Let Ω be a domain in $\mathbb{R}^N$ and consider a second order linear partial differential operator A in divergence form on Ω which is not required to be uniformly elliptic and whose coefficients are allowed to be complex, unbounded and measurable. Under rather general conditions on the growth of the coefficients we construct a quasi-contractive analytic semigroup $(e^{-tA_V})_{t\geqslant 0}$ on $L^2(\Omega, dx)$, whose generator A_V gives an operator realization of A with general boundary conditions. Under suitable additional conditions on the imaginary parts of the diffusion coefficients, we prove that for a wide class of boundary conditions, the semigroup $(e^{-tA_V})_{t\geqslant 0}$ is quasi-L^p-contractive for $p \in (1, \infty)$. We then show that the semigroup $(e^{-tA_V})_{t\geqslant 0}$ is a semigroup of integral operators. Our main result is pointwise Gaussian upper bounds for the integral kernel of $(e^{-tA_V})_{t\geqslant 0}$. In contrast to the previous literature the diffusions coefficients are not required to be bounded or regular. A new approach based on Davies-Gaffney estimates is used. It is applied to a number of examples, including some degenerate elliptic operators arising in Financial Mathematics, and generalized Ornstein-Uhlenbeck operators with potentials.

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No boundary conditions for wave equations on an interval

Delio Mugnolo
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Cosine operator functions

We consider one-dimensional wave equations subject to constraints on the mo- ments of order 0 and 1 of the unknown, instead of more common boundary condi- tions. This is studied by a combination of energy methods and Lord Kelvin's image principle. The relevant phase spaces turn out to be some space of distributions on the torus and the space of continuous function over the interval, respectively. This is joint work with Adam Bobrowski (Lublin, Poland) and Serge Nicaise (Valenciennes, France).

On joint numerical radius

Vladimir Müller **Special classes of operators in Banach and Hilbert spaces**
Academy of Sciences of the Czech Republic, Czech Republic

Let $T_1, \dots, T_n$ be bounded linear operators on a complex Hilbert space H. We study the question whether it is possible to find a unit vector $x \in H$ such that $|\langle T_j x, x \rangle|$ is large for all j. Thus we are looking for a generalization of a well-known fact for $n = 1$ that the numerical radius $w(T)$ of a single operator T satisfies $w(T) \geq \|T\|/2$.

Flow in networks with sinks

Proscovia Namayanja
University of KwaZulu-Natal, South Africa

Semigroups in natural sciences

In this talk, we show that the transport problem on a network is well-posed if and only if the network has no sinks. However, in the presence of sink components, the flow problem is well-posed. We explore other approaches that can be used to turn the ill-posed problem into a well-posed problem.

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Laplace transform inversion and approximaton of semigroups

Frank Neubrander **Approximation and perturbation of semigroups**
Louisiana State University, United States

In this report on joint work with Koray Özer and Lee Windsperger, we present Mathematicasupported proofs of error estimates for rational approximations of operator semigroups (i.e., numerically effective approximations of semigroups in terms of fnite sums of the resolvents of their generators) and their applications to Laplace transform inversion.

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Asymptotic behavior of a passive tracer in random fields

Ernest Nieznaj
Lublin University of Technology, Poland

Asymptotic behaviour of semigroups

We investigate the asymptotic behavior of trajectories of a passive tracer given by the solution of an ordinary differential equation

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}(\mathbf{x}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0,$$

where $\mathbf{F}(\mathbf{x})$ is a d-dimensional random field. We prove that for gaussian and Poisson field of shot noise type and certain conditions imposed on the energy spectrum of $\mathbf{F}$ the behavior of $\mathbb{E}|\mathbf{x}(t)|^2$, when $t \to +\infty$, is superdiffusive.

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Convergence of semigroups associated to heat propagation models

Andrzej Palczewski
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Asymptotic behaviour of semigroups

The subject of the talk is the analysis of different models of heat propagation. As is well known, one of essential disadvantages of the classical model proposed by Fourier is the infinite velocity with which heat propagates. In recent years several new models have been proposed which give finite velocity of heat waves but are parabolic in their character. All these models lead to singularly perturbed equations. We analyze some of these models and prove that the solution of the classical heat equation (Fourier model) is a bulk approximation to exact solutions of these models. The main tool in these proofs is the convergence of semigroups associated to corresponding models of heat propagation.

Feynman-Kac theorem in Hilbert spaces

Irina V. Melnikova
Ural Federal University, Russia

Semigroups in natural sciences

Valentina Parfenenkova
Ural Federal University, Russia

The relation between a solution $X = \{X(t), t \in [0,T]\}$ to the abstract stochastic Cauchy problem in Hilbert spaces U, H

$$dX(t) = AX(t)dt + BdW(t), \quad t \in [0,T],\ X(0) = y,\ B \in \mathcal{L}(U,H)$$

and a solution to the infinite dimensional deterministic partial differential Cauchy problem

$$\frac{\partial g}{\partial t}(t,x) = \frac{\partial g}{\partial x}(t,x)Ax + \frac{1}{2}Tr\left[B\frac{\partial^2 g}{\partial x^2}(t,x)B*\right], \quad t \in [0,T],\ g(0,x) = h(x), \qquad (1)$$

for the probability characteristic $g(t,x) = \mathbb{E}^{T-t,x}h(X(T))$ with some measurable function h from H to $\mathbb{R}$ is considered. Here A is the generator of a C_0–semigroup in H, and W is a U-valued Q-Wiener process.

The major aim is to present proofs of the relation based on two different approaches: based on usage of Ito's formila and based on usage of semigroup properties. "Ito" approach consists of at first proof of the Markov property for the Cauchy problem solution X, then the martingal property for the function $g(t,x)|_{x=X(t)}$ and at last formal usage of infinite dimensional Ito's formula to $g(t,X(t))$.

"Semigroup" approach is based on semigroup properties of the family of operators generated by the operator on right-side of the equation (1).

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Robustness of polynomial stability of semigroups

Lassi Paunonen
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Asymptotic behaviour of semigroups

In this presentation we consider a strongly continuous semigroup $T(t)$ generated by $A : \mathcal{D}(A) \subset X \to X$ on a Hilbert space X. The semigroup is called *polynomially stable* if $T(t)$ is uniformly bounded, if $i\mathbb{R} \subset \rho(A)$, and if there exist constants $\alpha > 0$ and $M > 0$ such that [1]

$$\|T(t)A^{-1}\| \leq \frac{M}{t^{1/\alpha}} \qquad \forall t > 0. \tag{1}$$

We are interested in the preservation of the polynomial stability of $T(t)$ under finite-rank perturbations $A + BC$ of its generator. In particular, we assume $B \in \mathcal{L}(\mathbb{C}^m, X)$ and $C \in \mathcal{L}(X, \mathbb{C}^m)$, and that for some $\beta, \gamma \geq 0$ the operators satisfy

$$\mathcal{R}(B) \subset \mathcal{D}((-A)^\beta) \qquad \text{and} \qquad \mathcal{R}(C^*) \subset \mathcal{D}((-A^*)^\gamma). \tag{2}$$

Under these assumptions $(-A)^\beta B$ and $(-A^*)^\gamma C^*$ are bounded operators.

The main result of the presentation is stated in the following theorem [2].

Theorem 1. *If $\beta + \gamma \geq \alpha$, then there exists $\delta > 0$ such that for all B and C satisfying* (2) *and $\|(-A)^\beta B\| \cdot \|(-A^*)^\gamma C^*\| < \delta$ we have $\sigma(A + BC) \subset \mathbb{C}^-$, the semigroup $T_{A+BC}(t)$ generated by $A + BC$ is uniformly bounded, and there exists $M > 0$ such that*

$$\|T_{A+BC}(t)(A + BC)^{-1}\| \leq \frac{M}{t^{1/\alpha}}, \qquad \forall t > 0.$$

In particular, the perturbed semigroup is strongly and polynomially stable.

The perturbation results have an application in robust output regulation of linear distributed parameter systems with infinite-dimensional exosystems.

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Long time behaviour of the stochastic model of stem cells differentiation with random switching

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Semigroups in biology/Markov semigroups

We investigate a piece-wise deterministic Markov process (PDMP) constructed from the deterministic model of stem cell differentiation. The deterministic model was presented by Anna Marciniak-Czochra in [1]. A crucial parameter for the stationary solutions of the deterministic model is a fraction of self-renewal. In [2] it is shown that the fraction of self-renewal is also a crucial parameter for the stationary solution of the stochastic Itó modification of the latter model. We modify the model by converting the parameter of the fraction of self-renewal from a constant parameter into a discrete Markov process. In this way we obtain a piece-wise deterministic Markov process. The main goal of this research is to investigate the long-time behaviour of the Markov semi-group related to the PDMP.

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Strong mixing measures for C_0-semigroups

Alfred Peris
Polytechnic University of Valencia, Spain

Linear models in chaotic dynamics

We will present a general method to prove that certain C_0-semigroups admit invariant strongly mixing measures. More precisely, the Frequent Hypercyclicity Criterion for C_0-semigroups ensures the existence of invariant mixing measures with full support. Our approach is different from Bayart and Matheron's [1] and Rudnicki's (see, e.g., [2]). We will give some examples, that range from birth-and-death models to the Black-Scholes equation, which illustrate these results. This is a joint work with Marina Murillo-Arcila.

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Self–similar asymptotics of solutions to heat equation with inverse square potential

Dominika Pilarczyk
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Heat kernels, Green's functions and Hardy spaces

We study properties of solutions to the initial value problem

$$u_t = \Delta u + \frac{\lambda}{|x|^2}u, \quad x \in \mathbb{R}^n, \quad t > 0$$
$$u(x,0) = u_0(x),$$

where $\lambda \in \mathbb{R}$ is a given parameter. We show, using the estimates of the fundamental solution, that the large time behavior of solutions to this problem is described by the explicit self-similar solutions.

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The discretization of Bitzadze-Samarsky type inverse problem for elliptic equations with Dirichlet and Neumann conditions

Sergey Piskarev
Lomonosov Moscow State University, Russia **Semigroups for evolution equations**

This talk is devoted to the numerical analysis of inverse problem for abstract elliptic differential equations with Bitzadze-Samarsky conditions. The presentation uses general approximation scheme and is based on C_0-semigroup theory and a functional analysis approach.

In the first part of talk we present results of [1]. In the second part of talk in a complex Banach space E we consider the problem of finding a function $u(\cdot) \in C^2([0,T];E) \cap C([0,T];D(A))$ and an element $\varphi \in E$ from the system

$$\begin{cases} u''(t) = Au(t) + \varphi, \quad 0 \leqslant t \leqslant T, \\ u'(0) = x, \\ u'(T) = \sum\limits_{i=1}^{L} k_i u'(\xi_i) + y, \\ u(\theta) = z, \end{cases} \tag{1}$$

where $\{\xi_i\}$ is the sequence of the various numbers in the interval $(0,T)$, the number $\theta \in (0,T)$ is fixed and the coefficients $\{k_i\}$ are real, A is a closed linear operator with dense domain $D(A)$ in the space E, the element $z \in D(A)$ is given.

One can consider the Neumann problems in Banach spaces E_n :

$$u_n''(t) = A_n u_n(t) + \varphi_n,\ t \in [0,T],\ u_n'(0) = \tilde{u}_n^0,\ u_n'(T) = \tilde{u}_n^T, \tag{2}$$

with strongly positive operators A_n, A_n and A are compatible, $u_n^0 \to u^0$, $\ u_n^T \to u^T$. We are going to describe here also the discretization of (2) in variable t. One of the simplest difference scheme is

$$\begin{aligned} &\frac{U_n^{k+1} - 2U_n^k + U_n^{k-1}}{\tau_n^2} = A_n U_n^k + \bar{\varphi}_n, \quad k \in \{1, ..., [\frac{T}{\tau_n}] - 1\}, \\ &U_n^1 - U_n^0 = \tau_n \tilde{u}_n^0,\ U_n^K - U_n^{K-1} = \tau_n \tilde{u}_n^T. \end{aligned} \tag{3}$$

Analysis of the methods (2) and (3) are given.

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On the reflexivity, hyperreflexivity and transitivity of Toeplitz operators

Marek Ptak **Special classes of operators in Banach and Hilbert spaces**
University of Agriculture, Kraków, Poland

The reflexivity, transitivity and hyperreflexivity results for subspaces and algebras of Toeplitz operators will be presented. We start with the classical result about reflexivity and hyperreflexivity of analytic Toeplitz operators on the Hardy space on the unit disc. The space of all Toeplitz operators is transitive but 2–reflexive. We will study the dichotomic behavior of subspaces of Toeplitz operators on the Hardy space. A linear space of Toeplitz operators which is closed in the ultraweak operator topology is either transitive or reflexive. No intermediate behavior is possible. This result can be extended to the Toeplitz operators on the Hardy space on the upper half–plane. The Toeplitz operators on the Bergman space will be also considered. The generalized Toeplitz and the multivariable Toeplitz operators case will be also considered.

Kernel estimates for nonautonomous Kolmogorov equations

Abdelaziz Rhandi
University of Salerno, Italy

Luca Lorenzi
University of Parma, Italy

Markus Kunze
University of Ulm, Germany

Semigroups for evolution equations

Using time dependent Lyapunov functions, we prove pointwise upper bounds for the heat kernels of some nonautonomous Kolmogorov operators with possibly unbounded drift and diffusion coeffcients. As an application we show that the kernel p of the evolution family generated by

$$(A(t)\varphi)(x) = (1 + |x|^m)Tr(Q^0(t,x)D^2\varphi(x)) - b(t,x)|x|^p x \cdot \nabla\varphi(x)$$

satisfies

$$0 < p_{t,s}(x,y) \leq (t-s)^{-\beta} e^{-\delta_0 (t-s)^\alpha} |y|^{p+1-m},\ t \in (0,1], s \in (0,t), x, y \in \mathbb{R}^d,$$

where $m \geq 0$, $p > \max\{m-1, 1\}$, $\alpha > (p+1-m)/(p-1)$ and δ_0, β are suitable positive constants. Here Q^0 and b are, respectively, a matrix valued function and a scalar function satisfying appropriate conditions. This generalizes the examples in [1] and [2].

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Piece-wise deterministic processes in biological models

Ryszard Rudnicki
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Semigroups in biology/Markov semigroups

In my talk I am going to present some biological processes modelled by means of piecewise-deterministic processes. We study stochastic semigroups corresponding to these processes. The main result is asymptotic stability of the involved semigroups in the set of densities. The strategy of the proof of this result is as follows. First we show that the transition function of the related stochastic process has a kernel (integral) part. Then we find a set E on which the density of the kernel part of the transition function is positive. Next we show that the set E is a stochastic attractor. Then we apply results concerning asymptotic behavior of partially integral stochastic semigroups. We show that the semigroup satisfies the "Foguel alternative", i.e. it is either asymptotically stable or "sweeping". If the attractor E is a compact set then the semigroup is asymptotically stable. We show how this method works analysing a gene expression model [1].

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Invariant sets for semigroups of nonlinear operators

Wolfgang Ruess
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Approximation and perturbation of semigroups

In the context of the Cauchy problem

$$\text{(CP)} \quad \begin{cases} \dot{u}(t) + Bu(t) \ni f(t, u(t)), & t \geq 0, \\ u(0) = u_0, \end{cases}$$

with $B \subset X \times X$ an accretive operator, the basic question is about criteria for invariance of a closed subset C of the state Banach space X under solutions to (CP): $u_0 \in C \Rightarrow u(t) \in C$ for all $t \geq 0$. While there are 'classical' results for this case by Amann, Bothe, Brézis/Browder, Crandall, Deimling, Nagumo and many others, the aim of this talk is to present results on the corresponding problem for partial differential delay problems of the form

$$\text{(PFDE)} \quad \begin{cases} \dot{u}(t) + Bu(t) \ni F(u_t), & t \geq 0 \\ u_{|I} = \varphi \in \hat{E}, \end{cases}$$

with $I = [-R, 0]$, or $I = (-\infty, 0]$, as well as for its nonautonomous version, with $B(t)$, $F(t; \cdot)$, and $\hat{E}(t)$ time-dependent.

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A global attractor of a sixth order Cahn-Hilliard type equation

Maciej Korzec
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Asymptotic behaviour of semigroups

Piotr Nayar
The University of Warsaw, Poland

Piotr Rybka
The University of Warsaw, Poland

We study a sixth order convective Cahn-Hilliard type equation type that describes the faceting of a growing surface. It is considered with periodic boundary conditions. We deal with the problem in one and two dimensions. We establish the existence and uniquness of weak solutions. We also show existence of global attractor in dimensions one and two.

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An evolution operator for the nonstationary Sobolev type equation

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Semigroups in natural sciences

Consider the nonstationary equation

$$L\dot{u}(t) = M_t u(t), \qquad t \in \mathfrak{J} \subset \mathbb{R} \tag{1}$$

where operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$, $M_t \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ for every $t \in \mathfrak{J}$. If $\ker L \neq \{0\}$ then (1) is called *Sobolev type equation* [1].

Definition 1. Sets $\rho^L(M_t) = \{\mu \in \mathbb{C} : (\mu L - M_t)^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{U})\}$ and $\sigma^L(M_t) = \mathbb{C} \setminus \rho^L(M_t)$ are called *L-resolvent set* and *L-spectrum* of operator-function M_t correspondingly.

The operator-function M_t is called *spectrally bounded with respect to operator* L (or simply (L,σ)*-bounded*), if

$$\exists a_t \in C(\mathfrak{J};\mathbb{R}_+) \quad \forall t \in \mathfrak{J} \quad \max\{|\mu| :\ \mu \in \sigma^L(M_t)\} \leq a_t < +\infty.$$

Let the operator-function M_t be (L,σ)-bounded and the contour $\gamma_t = \{\mu \in \mathbb{C} : |\mu| = 2a_t\}$. Consider integrals

$$P_t = \frac{1}{2\pi i} \int_{\gamma_t} R^L_\mu(M_t) d\mu, \qquad Q_t = \frac{1}{2\pi i} \int_{\gamma_t} L^L_\mu(M_t) d\mu.$$

Operators $P_t : \mathfrak{U} \to \mathfrak{U}$ and $Q_t : \mathfrak{F} \to \mathfrak{F}$ are projectors. It was proved in [1] with fixed $t \in \mathfrak{J}$.

Theorem 1. [2] *Let the operator-function $M_t \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ be (L,σ)-bounded. Then*

(i) *the action of operators $L_{t,k} : \mathfrak{U}^k_t \to \mathfrak{F}^k_t$, $M_{t,k} : \mathfrak{U}^k_t \to \mathfrak{F}^k_t$ $\forall t \in \mathfrak{J}$, $k = 0, 1$ is observed;*

(ii) *there exists an operator $M^{-1}_{t,0} \in \mathcal{L}(\mathfrak{F}^0_t;\mathfrak{U}^0_t)$, $t \in \mathfrak{J}$, besides if the operator-function $M_t : \mathfrak{J} \to \mathcal{L}(\mathfrak{U};\mathfrak{F})$ is strongly differential then the operator-function $M^{-1}_{t,0}(I - Q_t) \in \mathcal{L}(\mathfrak{F};\mathfrak{U}^0_t)$ is also strongly differential by $t \in \mathfrak{J}$ and if the operator-function $\frac{d}{dt}M_t$ is strongly continuous then the operator-function $\frac{d}{dt}(M^{-1}_{t,0}(I - Q_t))$ is also strongly continuous by $t \in \mathfrak{J}$;*

(iii) *there exists an operator $L^{-1}_{t,1} \in \mathcal{L}(\mathfrak{F}^1_t;\mathfrak{U}^1_t)$, $t \in \mathfrak{J}$ where the operator-function $L^{-1}_{t,1} Q_t \in C(\mathfrak{J};\mathcal{L}(\mathfrak{F};\mathfrak{U}^1_t)$.*

Definition 2. The (L,σ)-bounded operator-function M_t is called $(L,0)$*-bounded* if $\forall t \in \mathfrak{J}$ $M^{-1}_{t,0} L_{t,0} = H_t \equiv \mathbb{O}$.

Theorem 2. [2] *Let the operator-function $M_t \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ be $(L,0)$-bounded. Then* $\ker L = \mathfrak{U}^0_t$, $\operatorname{im} L = \mathfrak{F}^1_t$ *for all* $t \in \mathfrak{J}$.

Set $\ker L = \ker P_t = \mathfrak{U}^0$, $\ker Q_t = \mathfrak{F}^0_t$; $\operatorname{im} P_t = \mathfrak{U}^1_t$ and $\operatorname{im} L = \operatorname{im} Q_t = \mathfrak{F}^1$. By L_0 $(M_{t,0})$ denote the restriction of operator L (M_t) on $\mathfrak{U}^0$ and by $L_{t,1}$ $(M_{t,1})$ the restriction of operator L (M_t) on $\mathfrak{U}^1_t$, $t \in \mathfrak{J}$.

The vector-function $u \in C^1(\mathfrak{J};\mathfrak{U})$ satisfying (1) is called *the solution* of this equation on the $\mathfrak{J}$.

If the operator-function M_t is $(L, 0)$-bounded then we can gets the equation

$$\dot{f}(t) = M_t L_{t,1}^{-1} f(t)$$

with the operator-function $T_t = M_{t,1} L_{t,1}^{-1} \in C(\mathfrak{J}; \mathcal{L}(\mathfrak{F}^1))$. The solution for Cauchi problem $f(t_0) = f_0 \in \mathfrak{F}^1$ of this equation can be found [3] by the form $f(t) = \tilde{F}(t) f_0$ where *operator Cauchi*

$$\tilde{F}(t) = I_{\mathfrak{F}^1} + \int\limits_{t_0}^{t} T_{t_1} dt_1 + \sum_{n=2}^{\infty} \int\limits_{t_0}^{t} \int\limits_{t_0}^{t_n} \ldots \int\limits_{t_0}^{t_2} T_{t_n} T_{t_{n-1}} \ldots T_{t_1} dt_1 \ldots dt_n \in \mathcal{L}(\mathfrak{F}^1).$$

Definition 3. The operator $U(t, \tau) = L_{t,1}^{-1} \tilde{F}(t) \tilde{F}^{-1}(\tau) L_{\tau,1} P_\tau$ is called an *evolution* (*solving*) *operator* for (1).

Theorem 3. [2] *The evolution operator has the following properties:*

(i) $U(t, t) = P_t$;

(ii) $U(t, s) U(s, \tau) = U(t, \tau)$;

(iii) $U(t, \tau)\Big|_{\mathfrak{U}_\tau^1} = \left[U(\tau, t)\Big|_{\mathfrak{U}_t^1} \right]^{-1}$;

(iv) $\|U(t, \tau)\|_{\mathcal{L}(\mathfrak{U})} \leq K \exp \left(\int\limits_{\tau}^{t} \|T_s\|_{\mathcal{L}(\mathfrak{F}^1)} ds \right)$ $(\tau \leq t)$.

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Splitting methods for Schrödinger equations with singular potentials

Roland Schnaubelt **Approximation and perturbation of semigroups**
Karlsruhe Institute of Technology, Germany

We study the error analysis for time integration schemes for the linear Schrödinger equation

$$iu'(t) = -\Delta u(t) + Vu(t), \quad t \in \mathbb{R}, \qquad u(0) = u_0,$$

in $L^2(\mathbb{R}^d)$ with a real potential V. The structure of this equation suggests to use splitting methods for the numerical approximation of the solution $U(t)u_0$, where $i(\Delta - V)$ generates $U(\cdot)$. To this end, one solves the two much more simple equations

$$iv'(t) = -\Delta v(t), \qquad iw'(t) = Vw(t),$$

separately. There are very efficient numerical algorithms to approximate the respective unitary groups $T(\cdot)$ generated by $i\Delta$ and $S(\cdot)$ generated by iV. The products $[T(\frac{t}{n})S(\frac{t}{n})]^n u_0$, resp. $[S(\frac{t}{2n})T(\frac{t}{n})S\frac{t}{2n})]^n u_0$, should converge to $U(t)u_0$. For bounded potentials with bounded derivatives first, resp. second, order convergence was shown for $u_0 \in H^1$, resp. $u_0 \in H^2$ in the seminal paper [1]. For potentials with local singularities we establish analogous bounds with a reduced convergence order depending on the integrability properties of V and its derivatives. Our proofs use new formulas for the time discretization error and Strichartz' estimates. We focus on the time semi–discretization on the level of the partial differential equation.

This is joint work with Marlis Hochbruck and Tobias Jahnke (Karlsruhe).

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Strong convergence in L^p–spaces for invariant measures for non–autonomous Kolmogorov equations

Roland Schnaubelt
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Asymptotic behaviour of semigroups

We consider linear parabolic equations on $\mathbb{R}^d$ with unbounded time dependent diffusion and drift coefficients. The main assumption involves a so-called Lyapunov function for this problem which implies the existence of a family of invariant probability measures μ_t, see [1]. This means that

$$\int_{\mathbb{R}^d} U(t,s)\varphi \, d\mu_t = \int_{\mathbb{R}^d} \varphi \, d\mu_s =: m_s(\varphi)$$

for all $t \geq s \geq 0$ and bounded Borel functions φ, where $U(t,s)$ is the evolution family solving the parabolic equation. Then $U(t,s)$ can be extended to a conctraction from $L^p(\mu_s)$ to $L^p(\mu_t)$. Our main result says that $U(t,s)\varphi$ converges to the mean $m_s(\varphi)$ locally uniformly and in $L^p(\mu_t)$, as $t \to \infty$. A similar result holds as $s \to -\infty$. Our proofs rely on global gradient estimates for $U(t,s)$ from [1], classical local Schauder estimates and certain properties of the evolution semigroup associated with $U(t,s)$.

This is joint work with Luca Lorenzi and Alessandra Lunardi (Parma).

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Rates of decay in the classical Katznelson-Tzafriri theorem

David Seifert
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Asymptotic behaviour of semigroups

This talk will introduce the Katznelson-Tzafriri theorem for a single operator and then present some recent results, inspired by analogous developments in the theory of C_0-semigroups, which provide bounds on the rate at which decay takes place in the original result. These bounds are then shown, in an import special case, to be optimal on Banach space but not on Hilbert space.

Homogeneous Calderón-Zygmund estimates for a class of second order elliptic operators

Chiara Spina
University of Salento, Italy

Semigroups for evolution equations

Given a uniformly elliptic operator $L = \sum_{i,j=1}^{N} a_{ij}(x)D_{ij}$, with (a_{ij}) bounded and uniformly continuous (BUC) functions in $\mathbb{R}^N$, $N \geq 2$, a-priori estimates and solvability results in Sobolev spaces for the associated Poisson problem are well known in literature. In this context, a basic role is played by the classical inequality

$$\|D^2u\|_p \leq C(\|Lu\|_p + \|u\|_p), \quad u \in W^{2,p}$$

that leads, in addition, to the unique resolution of the resolvent equation $Lu - \lambda u = f$, $\lambda > 0$. We are interested, among other things, in establishing the *stronger* homogeneous estimate

$$\|D^2u\|_p \leq C\|Lu\|_p\,, \quad D^2u \in L^p\,. \tag{1}$$

To the best of our knowledge, results concerning the validity of (1) have been proved only in certain special cases.

We show that, under the assumptions that the $a_{ij}(x)$ are strongly elliptic, BUC and possess a limit as $|x| \to \infty$, for any given $f \in L^p$ equation $Lu = f$ has one and only one solution in *homogeneous Sobolev spaces* satisfying (1). On the other hand, we also exhibit an example which shows that if the condition of the existence of the limit is removed, then inequality (1) is not true. Thus, this condition is clearly pivotal for the validity of our result.

As a corollary to the above result, we are able to show the resolvent estimate

$$\|(\lambda - L)^{-1}f\|_p \leq \frac{C}{\lambda}\|f\|_p,$$

for any $\lambda > 0$, and with $C = C(p) > 0$.

Joint work with G.P. Galdi, G. Metafune, C. Tacelli.

Heat-type kernels: regularized traces and short-time asymptotics

Stanislav Stepin
University of Białystok, Poland

Heat kernels, Green's functions and Hardy spaces

An approach to the study of diffusion semigroups kernels based on the usage of Wiener path integral representation will be discussed. Within this approach explicit formulas for heat invariants are established and two-sided estimates for the heat trace are obtained. In the case of diffusion with a drift we make use of Feynman-Kac-Ito formula to specify short-time asymptotics. A semigroup generated by potential perturbation of biLaplacian is treated as a model in non-diffusion case. Parametrix expansion will be applied then to study short-time asymptotics of the corresponding integral kernel and its regularized trace.

Degenerate operator groups in the optimal measurement theory

Alexandr L. Shestakov
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Semigroups in natural sciences

Georgy A. Sviridyuk
South Ural State University, Russia

The optimal measurements theory (OMT) at first was intended to restore the distorted signals as mechanical inertia of the measurement transducer (MT) [1] and the resonances in his chains [2]. The basis of a mathematical model of MT is Leontieff type equations system

$$L\dot{x} = Mx + Du \tag{1}$$

and the Showalter – Sidorov initial condition

$$[R^L_\alpha(M)]^{p+1}(x(0) - x_0) = 0. \tag{2}$$

The second important component of the mathematical model of the MT is functional canceled J which in particular represents the difference between the signal $z = Cx$ results from (1), (2) and the signal z_0 received on the real measuring apparatus during the experiment. The reconstructed signal is the minimum point of the functional J on a closed and convex set $\mathfrak{U}_\partial$ of feasible optimal measurements. Numerical algorithm for finding of the optimal measurement uses the theory of degenerate operator groups [3]. The results can be applied to restore the signals corrupted by "white noise" [4]. The minimum of functional J is sought in spaces of "noise".

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Ergodic measures for Markov semigroups

Tomasz Szarek
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Asymptotic behaviour of semigroups

In joint work with D. Worm we study the set of ergodic measures for a Markov semigroup on a Polish state space. The principal assumption on this semigroup is the e property, an equicontinuity condition. We introduce a weak concentrating condition around a compact set K and show that this condition has several implications on the set of ergodic measures, one of them being the existence of a Borel subset K_0 of K with a bijective map from K_0 to the ergodic measures, by sending a point in K_0 to the weak limit of the Cesáro averages of the Dirac measure on this point. We also give sufficient conditions for the set of ergodic measures to be countable and finite. Finally, we give a quite general condition under which the Cesáro averages of any measure converge to an invariant measure.

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On Schrödinger operator with unbounded coefficients

Cristian Tacelli
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Semigroups for evolution equations

Joint work with A. Canale, A. Rhandi
University of Salerno, Italy

Let A be the Schrödinger type operator defined by

$$Au = a(x)\Delta u + V(x)u \ ,$$

where $a(x) = (1 + |x|^{\alpha})$ and $V(x) = -|x|^{\beta}$.

In the case $\alpha \in [0, 2]$ and $\beta > 0$ generation results of analytic semigroup in $L^p(\mathbb{R}^N)$ have been proved in [2] and estimates for the heat kernel are obtained. As regard the case $\beta = 0$ generation results and kernel estimates are obtained in [3] and [1].

We prove, for $\beta > \alpha - 2$ and $N > 2$, that the operator $(A, D_{p,max})$, where $D_{p,max} := \left\{u \in W^{2,p}_{loc}(\mathbb{R}^N) \cap L^p(\mathbb{R}^N) \mid Au \in L^p(\mathbb{R}^N)\right\}$, is invertible in $L^p(\mathbb{R}^N)$ for $1 < p < \infty$ obtaining the following potentials and gradient estimates

$$\|Vu\|_p \leq C\|Au\|_p \ \text{ for } \beta > \alpha - 2$$

and

$$\||x|^{\beta+1}\nabla u\|_p \leq C\|Au\|_p \text{ for } \beta > \alpha - 1$$

for every $u \in D_{p,\max}$.

Then, we prove that the realization A_p in $L^p(\mathbb{R}^N)$ with the maximal domain $D_{p,max}$ generates an analytic semigroup.

Finally, spectral properties of A and estimates for the heat kernel k associated to the semigroup $(T(t))_{t\geq 0}$ are obtained.

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The dynamics of enzyme inhibition controlled by piece-wise deterministic Markov process

Andrzej Tomski
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Semigroups in biology/Markov semigroups

Enzymes are the molecules (mainly the proteins) working in the cells as highly specialized catalysts of many biological processes. The molecules that decrease enzymes activity are called inhibitors. Currently, the inhibitors are well known not only for being a part of natural metabolic pathways in the organism, but also because of wide applications in pharmacology and biochemistry. In this talk I will present a model of enzyme inhibition as an example of piece-wise deterministic Markov process. Long-time behavior of densities of the process will be discussed. I will also recall the conditions under which the Foguel alternative for the corresponding Markov semigroup is satisfied. Finally, I will reveal the answer to the question: is this semigroup always asymptotically stable?

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Optimal polynomial decay via interplay between semigroup

Roberto Triggiani **Plenary Talk**
University of Memphis, United States

We shall focus at first on a simplified model of heat-structure interaction. Semigroup/functional analytic/elliptic theory produce optimal decay of all terms required except one. Optimal estimate for the latter is obtained by an ad hoc microlocal argument.

This is joint work with George Avalos and Irena Lasiecka. G.Avalos's talk will include the pressure term for the fluid (linearized Navier-Stokes equation).

Heat kernel asymptotics on affine buildings

Bartosz Trojan
University of Wrocław, Poland

Heat kernels, Green's functions and Hardy spaces

Let $\mathcal{X}$ be a thick affine building of rank r. We consider a finite range isotropic random walk on vertices of $\mathcal{X}$. Our main focus is to obtain the optimal global upper and lower bounds for the n-th iteration of the transition operator.

The continuous counterpart of $\mathcal{X}$ is a Riemannian symmetric space of noncompact type. There the kernel of the heat semigroup for Laplace–Beltrami operator is well understood. The main results were obtained by Anker and Ji [1]. In [3] Guivarc'h, Ji and Taylor based on [1] constructed Martin compactification. The authors emphasize the importance of generalizations to Bruhat–Tits buildings associated with reductive groups over p-adic fields all the compactification procedures. Among Open Problems the asymptotic behaviour of the Green function of finite range isotropic random walks on affine buildings is formulated.

We show sharp lower and upper estimates on $p_n(x)$ uniform in the region

$$\operatorname{dist}(\delta, \partial\mathcal{M}) \geq K n^{-1/(2\eta)}$$

where $x \in V_\omega(O)$, $\delta = (n+r)^{-1}(\omega+\rho)$ and $\mathcal{M}$ is the convex envelop of the support of $p(x)$. Here, we state a variant of the result convenient in most applications

Theorem. For $\epsilon > 0$ small enough

$$p_n(x) \asymp n^{-r/2-|\Phi_0^+|}\rho^n e^{-n\phi(n^{-1}\omega)} P_\omega(0)$$

uniformly on $\{x \in V_\omega(x) \cap \operatorname{supp} p_n : \operatorname{dist}(n^{-1}\omega, \partial\mathcal{M}) \geq \epsilon\}$.

In the Theorem ρ is the spectral radius of p, P_ω Macdonald symmetric polynomial and $|\Phi_0^+|$ the number of positive root directions. The function ϕ is convex and satisfies $\phi(\delta) \asymp \|\delta\|^2$. If we denote by κ the spherical Fourier transform of p we can describe the asymptotic behaviour of the Green function

Theorem. $($i) If $\zeta \in (0, \rho^{-1})$ then for all $x \neq y$

$$G_\zeta(x,y) \asymp P_\omega(0)\|\omega\|^{-(r-1)/2-|\Phi_0^+|} e^{-\langle s,\omega\rangle}$$

where $y \in V_\omega(x)$ and s is the unique point such that $\kappa(s) = (\zeta\rho)^{-1}$ and

$$\frac{\nabla\kappa(s)}{\|\nabla\kappa(s)\|} = \frac{\omega}{\|\omega\|}.$$

(ii) If $\zeta = \rho^{-1}$ then for all $x \neq y$

$$G_\zeta(x,y) \asymp P_\omega(0)\|\omega\|^{2-r-2|\Phi_0^+|}$$

where $y \in V_\omega(x)$.

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A weak Gordon type condition for absence of eigenvalues of one-dimensional Schrödinger operators

Hendrik Vogt
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Approximation and perturbation of semigroups

Christian Seifert
Hamburg University of Technology, Germany

We show absence of eigenvalues for one-dimensional Schrödinger operators $-\Delta+\mu$ under the condition that the measure μ can be approximated by periodic measures in a suitable sense. Roughly speaking, we require that there are arbitrarily large periods $p > 0$ such that the three "pieces" $\mathbb{1}_{[-p,0]}\mu$, $\mathbb{1}_{[0,p]}\mu$ and $\mathbb{1}_{[p,2p]}\mu$ look very similar. This type of study is motivated by models of quasicrystals, where the corresponding potential is locally close to being periodic.

The important new aspect is that the distance of the three pieces is measured in a Wasserstein type metric and not in the total variation metric as in previous results. For linear combinations of Dirac measures this means that not only the coefficients but also the positions of the Dirac deltas are allowed to vary. Thus, in models of quasicrystals, the positions of atoms may be slightly perturbed from a quasiperiodic lattice.

Perturbations for linear delay equations in L_p

Jürgen Voigt **Approximation and perturbation of semigroups**
Technical University of Dresden, Germany

In the Cauchy problem for the linear delay equation

$$\begin{cases} u'(t) = Au(t) + Lu_t \quad (t \geq 0), \\ u(0) = x, \quad u_0 = f, \end{cases} \tag{DE}$$

with initial values $x \in X$, $f \in L_p(-h, 0; X)$ (where X is a Banach space, $1 \leq p < \infty$, and $0 < h \leq \infty$), the operator L is responsible for describing the influence of the 'past' on the evolution of the system. Traditionally, it is assumed that L is associated with a function $\eta\colon [-h, 0] \to \mathcal{L}(X)$ of bounded variation. In this case the problem (DE) can be treated for any $p \in [1, \infty)$. We present more general operators L that allow this treatment only for p in a proper subset of $[1, \infty)$: We require $L\colon W_p^1(-h, 0; X) \to X$ to be continuous as an operator from $L_r(\mu_L; X)$ to X, for some $r \in [1, p]$ and a suitable measure μ_L on $[-h, 0]$.

The talk is a report on joint work with H. Vogt.

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Of honesty theory and stochastic completeness

Chin Pin Wong
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Approximation and perturbation of semigroups

An important aspect in the study of Kato's perturbation theorem for substochastic semi-groups is the study of the honesty of the perturbed semigroup, i.e. the consistency between the semigroup and the modelled system. In the study of Laplacians on graphs, there is a corresponding notion of stochastic completeness. This talk will demonstrate how the two notions coincide.

Null controllable systems with vanishing energy

Jerzy Zabczyk
Polish Academy of Sciences, Poland

Semigroups of operators in control theory

The talk is concerned with infinite dimensional, linear, control systems. Conditions are presented under which arbitrary state can be transferred to the origin with arbitrarily small energy. The energy of a control is defined as its L-square norm. Both classical and boundary control system are considered. Abstract results are illustrated with specific examples.

The presentation is based on joint works with L. Pandolfi and E. Priola.

The degenerate operator groups theory and multipoint initial-finish problem for Sobolev type equations

Sophiya A. Zagrebina
South Ural State University, Russia

Semigroups in natural sciences

In [1] there was firstly introduced in consideration a degenerate group of operators $U^t = (2\pi i)^{-1} \int\limits_\Gamma R_\mu^L(M) e^{\mu t} d\mu$ as the resolution group of linear Sobolev type equation

$$L\dot{u} = Mu. \tag{1}$$

Here the operators $L,\ M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ and it is possible that $\ker L \neq \{0\}$, $R_\mu^L(M) = (\mu L - M)^{-1}L$; the contour $\Gamma \subset \mathbb{C}$ limits the domain that contains the L-spectrum $\sigma^L(M)$ of the operator M.

Then in [2] it was shown that $u(t) = U^t u_0$ is the unique solution of the Showalter – Sidorov problem

$$[R_\alpha^L(M)]^{p+1}(u(0) - u_0) = 0 \tag{2}$$

for the equation (1) for any $u_0 \in \mathfrak{U}$. Finally, in [3], [4] there was formulated and discussed the initial-finish problem for the equations of the form (1) which generalizes the problem (2). The first review of the initial-finish problems is given in [5].

The report discusses the basics of the theory of multipoint initial-finish problems for equations of the form (1) where the operator M is (L, p)-bounded. The sufficient conditions for the unique solvability are given. As an application we consider a multipoint initial-finish problem for the linear Oskolkov equations defined on a finite connected directed geometric graph. This problem is modelling the linear approximation of pumping of highly paraffinic sorts of oil.

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An alternative approximation of the degenerate strongly continuous operator semigroup

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Approximation and perturbation of semigroups

Inheriting and continuing the tradition, dating back to the Hill–Iosida–Feller–Phillips–Miyadera theorem, the new way of construction of the approximations for strongly continuous operator semigroups with kernels is suggested in the framework of the Sobolev type equations theory, which experiences an epoch of blossoming. We introduce the concept of relatively radial operator, containing the condition in the form of estimates for the derivatives of the relative resolvent. The existence of C_0-semigroup on some subspace of the original space is shown, the sufficient conditions of its coincidence with the whole space are given. The results are very useful in numerical study of different nonclassical mathematical models considered in the framework of the theory of the first order Sobolev type equations [1], and also to spread the ideas and methods to the higher order Sobolev type equations [2].

Let $\mathcal{U}$ and $\mathcal{F}$ be Banach spaces, operators $L \in \mathcal{L}(U;F)$ and $M \in \mathcal{C}l(U;F)$, function $f(\cdot) : \mathbb{R} \to \mathcal{F}$. Consider the Cauchy problem

$$u(0) = u_0 \tag{1}$$

for the operator-differential equation

$$L\,\dot{u} = Mu + f. \tag{2}$$

Following [1, 3], introduce the L-resolvent set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(F;U)\}$ and the L-spectrum $\sigma^L(M) = \overline{\mathbb{C}} \setminus \rho^L(M)$ of M. The operator functions $(\mu L - M)^{-1}$, $R^L_\mu(M) = (\mu L - M)^{-1}L$, $L^L_\mu(M) = L(\mu L - M)^{-1}$ are called L-resolvent, right and left L-resolvents of M.

Definition 1. The operator M is called *radial with respect to* L (shortly, L-radial), if
(i) $\exists a\in \mathbb{R} \quad \forall \mu > a \quad \mu\in \rho^L(M)$
(ii) $\exists K > 0 \quad \forall \mu > a \quad \forall n\in \mathbb{N}$

$$\max\{\|\frac{1}{n!}\frac{d^n}{d\mu^n}R^L_\mu(M)\|_{\mathcal{L}(U)}, \|\frac{1}{n!}\frac{d^n}{d\mu^n}L^L_\mu(M)\|_{\mathcal{L}(F)}\} \le \frac{K}{(\mu-a)^{n+1}}$$

Remark 1. Without loss of generality one can put $a = 0$ in definiton 1.

Set $\mathcal{U}^0 = \ker L \quad \mathcal{F}^0 = \ker L^L_\mu(M)$. By $L_0 \quad (M_0)$ denote restriction of $L \quad (M)$ to lineal $\mathcal{U}^0 \quad (\mathrm{dom} M_0 = \mathcal{U}^0 \cap \mathrm{dom} M)$.

By $\mathcal{U}^1 \quad (\mathcal{F}^1)$ denote the closure of the lineal $\mathrm{im}\, R^L_\mu(M) \quad (\mathrm{im}\, L^L_\mu(M))$ by norm of $\mathcal{U} \quad (\mathcal{F})$.

By $\tilde{\mathcal{U}}$ $(\tilde{\mathcal{F}})$ denote the closure of the lineal $\mathcal{U}^0 \dot{+}$ im $R^L_\mu(M)$ $(\mathcal{F}^0 \dot{+}$ im $L^L_\mu(M))$ by norm of $\mathcal{U}$ $(\mathcal{F})$. Obviously, $\mathcal{U}^1$ $(\mathcal{F}^1)$ is the subspace in $\tilde{\mathcal{U}}$ $(\tilde{\mathcal{F}})$.

Consider two equivalent forms of (2)

$$R^L_\alpha(M)\dot{u} = (\alpha L - M)^{-1} M u, \tag{3}$$

$$L^L_\alpha(M)\dot{f} = M(\alpha L - M)^{-1} f \tag{4}$$

as concrete interpretations of the equation

$$A\dot{v} = Bv, \tag{5}$$

defined on a Banach space $\mathcal{V}$, where the operators $A, B \in \mathcal{L}(V)$

Definition 2. The vector-function $v \in C(\overline{\mathbb{R}_+}; \mathcal{V})$, differentiable on $\mathbb{R}_+$ and satisfying (5) is called *a solution* of (5).

A little away from the standard [4], following [3] define

Definition 3. The mapping $V^\cdot \in C(\overline{\mathbb{R}_+}; \mathcal{L}(V))$ is called *a semigroup of the resolving operators* (a resolving semigroup) of (5), if

(i) $V^s V^t v = V^{s+t} v$ for all $s, t \geq 0$ and any v from the space $\mathcal{V}$;

(ii) $v(t) = V^t v$ is a solution of the equation (5) for any v from a dense in $\mathcal{V}$ set.

The semigroup is called *uniformly bounded*, if

$$\exists C > 0 \quad \forall t \geq 0 \quad \|V^t\|_{\mathcal{L}(V)} \leq C.$$

Theorem 1. *Let M be L-radial. Then there exists a uniformly bounded and strongly continuous resolving semigroup of (3) ((4)), treated on the subspace $\tilde{\mathcal{U}}$ $(\tilde{\mathcal{F}})$, presented in the form:*

$$U^t = s - \lim_{k \to +\infty} \frac{(-1)^{k-1}}{(k-1)!} \left(\frac{k}{t}\right)^k \left(\frac{d^{k-1}}{d\mu^{k-1}} R^L_\mu(M)\right)\Bigg|_{\mu = \frac{k}{t}},$$

$$(F^t = s - \lim_{k \to +\infty} \frac{(-1)^{k-1}}{(k-1)!} \left(\frac{k}{t}\right)^k \left(\frac{d^{k-1}}{d\mu^{k-1}} L^L_\mu(M)\right)\Bigg|_{\mu = \frac{k}{t}}).$$

The semigroup $\tilde{U}^t$ $(\tilde{F}^t)$ at first is defined not on the whole space $\mathcal{U}$ $(\mathcal{F})$, but on some subspace $\tilde{\mathcal{U}}$ $(\tilde{\mathcal{F}})$. Introduce the sufficient condition of their coincidence: $\mathcal{U} = \tilde{\mathcal{U}}$ $(\mathcal{F} = \tilde{\mathcal{F}})$.

Theorem 2. [1] *Let the space $\mathcal{U}$ $(\mathcal{F})$ be reflexive, the operator M be L-radial. Then $\mathcal{U} = \mathcal{U}^0 \oplus \mathcal{U}^1$ $(\mathcal{F} = \mathcal{F}^0 \oplus \mathcal{F}^1)$.*

References

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Phenotypic evolution of hermaphrodites

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Semigroups in biology/Markov semigroups

We consider finite, phenotype-structured population of hermaphrodites, and build an individual based model which describes interactions between the individuals. The model contains such elements as mating of individuals, inheritance of phenotypic traits, intraspecific competition and mortality. Here offspring's phenotype depends on traits of couple of parents, what constitutes some kind of novelty in individual based modeling, because at out knowledge there is no such a sexual model, while asexual ones are often studied in the literature (see e.g. [2]). We consider the limit passage with the number of individuals to infinity, what leads us to continuous distribution of phenotypic traits in the population. The model is described by partial differential equation, which contains nonlinear operators. The first of the operators is in charge of mating of individuals and inheritance, the other corresponds to the competition. We study two types of mating. The first one is random and is well-known in classical genetics, the second is assortative: the individuals mate more often with prototypically similar members of the population (see e.g. [1]).

The limiting version of the model is an evolutionary equation, containing bilinear operator. The particular case of the equation is Tjon-Wu equation which appears in the description of the energy distribution of colliding particles. In the case of random mating, under suitable conditions we prove the asymptotic stability result: distribution of the phenotypic traits in the population converges to stationary distribution. As a by-product we obtain relatively easy proof of Lasota-Traple theorem (see [3]) concerning asymptotic stability of Tjon-Wu equation. Moreover, we show applications of our theorem to some biologically reasonable situations of phenotypic inheritance.

References

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Speakers