Chiara Spina
University of Salento (Lecce), Italy

Homogeneous Calderón-Zygmund estimates for a class of second order elliptic operators

Given a uniformly elliptic operator $L = \sum_{i,j=1}^{N} a_{ij}(x) D_{ij}$, with (a_{ij}) bounded and uniformly continuous (BUC) functions in $\mathbb{R}^N$, $N \geq 2$, a-priori estimates and solvability results in Sobolev spaces for the associated Poisson problem are well known in literature. In this context, a basic role is played by the classical inequality

$$\|D^2 u\|_p \leq C(\|Lu\|_p + \|u\|_p), \quad u \in W^{2,p}$$

that leads, in addition, to the unique resolution of the resolvent equation $Lu - \lambda u = f$, $\lambda > 0$. We are interested, among other things, in establishing the *stronger* homogeneous estimate

$$\|D^2 u\|_p \leq C\|Lu\|_p\,, \quad D^2 u \in L^p\,. \tag{1}$$

To the best of our knowledge, results concerning the validity of (1) have been proved only in certain special cases.

We show that, under the assumptions that the $a_{ij}(x)$ are strongly elliptic, BUC and possess a limit as $|x| \to \infty$, for any given $f \in L^p$ equation $Lu = f$ has one and only one solution in *homogeneous Sobolev spaces* satisfying (1). On the other hand, we also exhibit an example which shows that if the condition of the existence of the limit is removed, then inequality (1) is not true. Thus, this condition is clearly pivotal for the validity of our result.

As a corollary to the above result, we are able to show the resolvent estimate

$$\|(\lambda - L)^{-1} f\|_p \leq \frac{C}{\lambda} \|f\|_p,$$

for any $\lambda > 0$, and with $C = C(p) > 0$.

Joint work with G.P. Galdi, G. Metafune, C. Tacelli.