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Phenotypic evolution of hermaphrodites

We consider finite, phenotype-structured population of hermaphrodites, and build an individual based model which describes interactions between the individuals. The model contains such elements as mating of individuals, inheritance of phenotypic traits, intraspecific competition and mortality. Here offspring's phenotype depends on traits of couple of parents, what constitutes some kind of novelty in individual based modeling, because at out knowledge there is no such a sexual model, while asexual ones are often studied in the literature (see e.g. [2]). We consider the limit passage with the number of individuals to infinity, what leads us to continuous distribution of phenotypic traits in the population. The model is described by partial differential equation, which contains nonlinear operators. The first of the operators is in charge of mating of individuals and inheritance, the other corresponds to the competition. We study two types of mating. The first one is random and is well-known in classical genetics, the second is assortative: the individuals mate more often with prototypically similar members of the population (see e.g. [1]).

The limiting version of the model is an evolutionary equation, containing bilinear operator. The particular case of the equation is Tjon-Wu equation which appears in the description of the energy distribution of colliding particles. In the case of random mating, under suitable conditions we prove the asymptotic stability result: distribution of the phenotypic traits in the population converges to stationary distribution. As a by-product we obtain relatively easy proof of Lasota-Traple theorem (see [3]) concerning asymptotic stability of Tjon-Wu equation. Moreover, we show applications of our theorem to some biologically reasonable situations of phenotypic inheritance.

References

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