International Conference Beyond Uniform Hyperbolicity, Będlewo, May 27 – June 7, 2013

Abstracts of talks

Robust non-uniform hyperbolicity

Martin Andersson

(Joint work with C. Vasquez)

It has been realized for some time now that open sets of dissipative diffeomorphisms with abundance of negative Lyapunov exponents can be obtained by combining partial hyperbolicity with a cleverly chosen notion of non-uniform hyperbolicity. These systems, baptized "partially hyperbolic diffeomorphisms whose central direction is mostly contracting", are well understood from the point of view of physical measures and statistical stability, and have been studied by a number of authors including Bonatti, Viana, Dolgopyat, de Castro, Yang, and myself. It turns out to be a much harder problem to identify and describe open sets of diffeomorphisms presenting non-trivial instances of positive Lyapunov exponents or with simultaneously positive and negative ones. This talk is about some new developments on that subject.

Breakdown of linear response for smooth families of dynamical systems with bifurcations

Viviane Baladi

(Joint with: M. Benedicks and D. Schnellmann)

Many interesting dynamical systems possess a unique SRB ("physical") measure, which behaves well with respect to Lebesgue measure. Given a smooth one-parameter family of dynamical systems f_t, is natural to ask whether the SRB measure depends smoothly on the parameter t. If the f_t are smooth hyperbolic diffeomorphisms (which are structurally stable), the SRB measure depends differentiably on the parameter t, and its derivative is

given by a "linear response" formula (Ruelle, 1997). When bifurcations are present and structural stability does not hold, linear response may break down. This was first observed for piecewise expanding interval maps, where linear response holds for tangential families, but where a modulus of continuity $t \log t$ may be attained for transversal families (Baladi-Smania, 2008). The case of smooth unimodal maps is much more delicate. Ruelle (Misiurewicz case, 2009) and Baladi-Smania (slow recurrence case, 2012) obtained linear response for fully tangential families (confined within a topological class). We present our new results on the transversal smooth unimodal case (including the quadratic family), where we obtain Holder upper and lower bounds (in the sense of Whitney, along suitable classes of parameters).

Structural stability of the inverse limit of endomorphisms

Pierre Berger

We prove that every endomorphism which satisfies Axiom A and the strong transversality conditions is C^1-inverse limit structurally stable. By endomorphism we mean a C^1-map of a compact manifold, which is possibly non-bijective and might have points at which the differential is not onto. The inverse limit of an endomorphism f is the space $\{(x_i)_i \in M^{\mathbb{Z}} : x_{i+1} = f(x_i)\}$. An endomorphism f is Axiom A if its nonwandering set is hyperbolic and equal to the closure of the periodic points. Its satisfies the strong transversality condition if for every preorbit $(x_i)_{i \leqslant 0}$ in a neighborhood of the nonwandering set has its associated local unstable manifold which is sent by f transversally to any local stable manifolds of the nonwandering set. We show that these conditions imply that every C^1-perturbation f' of f has its inverse limit homeomorphic to the one of f via a homeomorphism which conjugates the shift dynamics induced by f' and f. These conditions were also conjectured to be sufficient. This result is applied to the study of unfolding of some homoclinic tangencies. This also achieves a characterization of C^1-inverse limit structurally stable coverings.

Non uniformly hyperbolic diffeormorphisms derived from the standard map

Pablo Carrasco

(Joint work with Pierre Berger)

In spite of the highly developed techniques available in Pesin's theory of non-uniformly hyperbolic systems, not many non trivial examples are known, especially if one insist them to be robust. Even for apparently simple systems there appear to be major difficulties to establish the non-vanishing of the Lyapunov exponents. The very well known standard family $(s_r)_{r \in \mathbf{R}}$ of maps in the torus fits in the later category. It is conjectured that for a

positive Lebesgue set of parameters the corresponding map is non-uniformly hyperbolic on a positive area set (equivalently its metric entropy with respect to Lebesgue is positive). No single parameter is known where the conjecture holds. In this work we consider couplings of the standard map with a fast hyperbolic diffeomorphism. We prove that the resulting system is C^2 Robustly Non Uniformly Hyperbolic. In doing so we obtain several techniques that could be used to prove the non-vanishing of Lyapunov exponents for certain classes of partially hyperbolic diffeomorphisms, and also to study the properties of the standard map. Incidentally, we also obtain a counterexample in C^2 category to Bochi-Viana's results stating that C^1 generically a symplectomorphism is uniformly hyperbolic or Lebesgue a.e.(x) has zero as Lyapunov exponent with multiplicity two or more.

Phase transitions in the quadratic family

Daniel Coronel

(Joint work with Juan Rivera-Letelier)

We give the first examples of transitive quadratic maps having a phase transition after the first zero of the geometric (real and complex) pressure function. More precisely, we show two examples of phase transitions, one of first order and the other of higher order. In each example the quadratic map has a non-recurrent critical point, so it is non-uniformly hyperbolic in a strong sense.

Density and stability of ergodicity: partially hyperbolic diffeomorphisms

Sylvain Crovisier

(Joint work with Artur Avila and Amie Wilkinson)

We prove that stably ergodic diffeomorphisms are C^1-dense among volume-preserving partially hyperbolic C^2-diffeomorphisms, with any central dimension. This based on perturbations of horseshoes with large entropy.

Topological pressure as entropy of an extension

Tomasz Downarowicz

I will show that topological pressure associated with a nonnegative upper semicontinuous subadditive potential defined on a topological dynamical systems (X, T) equals the topological entropy of a special topological extension of (X, T). This extension is obtained as

a joining with a symbolic system. This sheds a new light on the notions of potential and topological pressure, giving it an information-theoretic interpretation.

Dynamical coherence and intrinsic ergodicity for partially hyperbolic diffeomorphisms isotopic to Anosov

Todd Fisher

(Joint work with Rafael Potrie and Martin Sambarino)

We discuss partially hyperbolic diffeomorphisms that are isotopic to a hyperbolic toral automorphism and contained in a connected component. If the splitting of the partially hyperbolic diffeomorphism satisfies certain dimensional constraints, then we show the diffeomorphism is dynamically coherent. We then prove that if the center direction is one dimensional, then the topological entropy is locally constant and there is a unique measure of maximal entropy.

Rigorous computation of invariant measure with small errors in the wasserstein distance

Stefano Galatolo

We describe an algorithm which is able to approximate invariant measures of dynamical systems up to small errors in the Wasserstein distance and its practical implementation. The use of Wasserstein distance, allow to replace some difficult a priori estimations on the regularity of the invariant measure and exploit as much as possible some a posteriori estimation which is made by the computer. The algorithm can hence be applied to systems which are not hyperbolic, as the Manneville map. We compute the invariant measure and the entropy of an example of such kind of maps up to small errors.

Accessibility and ergodicity of partially hyperbolic regular endomorphisms with 1D center bundle

Baolin He

We generalize Didier' and RRU' results about accessibility and ergodicity, to the case of partially hyperbolic regular endomorphisms with 1D center bundle.For partially hyperbolic regular endomorphisms with 1D center bundle,accessibility implies stable accessibility.In the set of partially hyperbolic regular endomorphisms with 1D center bundle, we show that

accessibility implies ergodicity, and that there is an C^r open and dense subset such that each is accessible. Here, the stable/unstable bundle can be trivial.

Beyond Kuperberg flows

Steve Hurder

The study of the dynamics of the celebrated Kuperberg flows on 3 manifolds reveals a geometric reason for the observation that they have entropy zero, due to a theorem of Katok. We show that these flows have a type of chaotic behavior, which results in their having positive "slow entropy". Moreover, we show that there are C^1-close flows where this chaotic behavior is due to the presence of horseshoe-like structures in the dynamics, which results in positive entropy in the usual sense. Thus, the Kuperberg flows lie at the boundary of hyperbolicity in an appropriate sense.

Convergence of fast-slow deterministic systems to a stochastic differential equation

Ian Melbourne

We describe recent work with Andrew Stuart, Georg Gottwald, and David Kelly, which gives rigorous proofs of convergence of fast-slow deterministic systems to a stochastic differential equation (SDE). Unlike other approaches that rely heavily on rapid (stretched exponential) decay of correlations for the fast dynamics which is notoriously hard to prove, the approach here depends on functional central limit theorems and large deviation estimates that are widely available for nonuniformly hyperbolic systems.

Dynamical Markov and Lagrange spectra for geodesic flows in surfaces with negative curvature

Carlos Gustavo Moreira

(Joint work with Sergio Romaña Ibarra)

Let φ be a diffeomorphism of a surface M and Λ a horseshoe associated to φ. Given a continuous function $f : M \to \mathbb{R}$, we define the *dynamical Lagrange spectrum* by

$$L(f, \Lambda) = \left\{ \limsup_{n\to\infty} f(\varphi^n(x)) : x \in \Lambda \right\},$$

and the *dynamical Markov spectrum* by

$$M(f,\Lambda) = \left\{ \sup_{n\in\mathbb{Z}} f(\varphi^n(x)) : x \in \Lambda \right\}.$$

We prove that typically, if the Hausdorff dimension of Λ is larger than 1, then for a (large) set of "typical" functions $f \in C^1(M,\mathbb{R})$, their dynamical Markov and Lagrange spectra have persistently non-empty interior.

Let M be a manifold, and let X be a complete vector field over SM . We define the dynamical Lagrange and Markov spectra associated to the pair (f,X), where $f \in C^0(M,\mathbb{R})$ by

$$L(f,X) = \left\{ \limsup_{n\to\infty} f(X^t(x)) : x \in M \right\},$$

and

$$M(f,X) = \left\{ \sup_{t\in\mathbb{R}} f(X^t(x)) : x \in M \right\}$$

respectively, where $X^t(x)$ is the trajectory of the flow of X beginning in x.

We prove that if M is a surface with a Riemannian metric g with negative curvature, bounded between two negative constants, and finite volume, and SM is the unit tangent bundle of M then there is a metric g_0 which may be taken arbitrarily close to the original metric g, such that the Markov and Lagrange dynamical spectra associated to any field close enough to the geodesic field in SM of the metric g_0 has persistently non-empty interior for an open and dense set of functions in $C^2(SM,\mathbb{R})$.

Invariant distributions for circle diffeomorphisms and a equidistribution theorem a la Weyl/Herman

Andres Navas

We show that circle diffeomorphisms in the Denjoy class and having irrational rotation number carry no invariant 1-distribution other than the invariant measure. As a corollary, we show an equidistribution theorem for smooth potentials. Using this, we retrieve in a conceptual way a theorem of M. Herman concerning convergence to the identity of certain iterates of C^2 circle diffeomorphisms. (This is joint work with M. Triestino, closely related to a work of Avila and Kocsard.) Generalizations for groups of diffeomorphisms will also be proposed.

Kupka-Smale systems at the boundary of uniform hyperbolicity

Isabel Rios

We discuss some examples of diffeomorphisms at the boundary of uniform hyperbolicity, stating some geometric and ergodic properties.

Equilibrium states of one-dimensional maps under a weak hyperbolicity assumption

Juan Rivera-Letelier

(Joint work with Huaibin Li)

For a real or complex one-dimensional map satisfying a weak hyperbolicity assumption, we describe the thermodynamic formalism for Holder continuous potentials. We show that, rather unexpectedly, such a map has no phase transitions: The pressure function is real analytic on the space of Holder continuous potentials, and every Holder continuous potential has a unique equilibrium state. This last fact, together with various formulas to compute the pressure function, allows us to apply Kifer's method to obtain a full level-2 large deviation principle for periodic points, iterated preimages, and Birkhoff averages.

Law of iterated logarithm for the turning point of tent maps

Daniel Schnellmann

We show that for almost every map in a non-degenerate one-parameter family of tent maps the Birkhoff sum of suitable observables along the forward orbit of the turning point satisfies the law of iterated logarithm. In fact we will prove an almost sure invariance principle for the Birkhoff sum (as a function on the parameter space).

Mono-atomic disintegration of volume along central foliation of derived from Anosov diffeomorphisms

Ali Tahzibi

(Joint work with G. Ponce and R. Varao)

We mainly address the problem of disintegration of Lebesgue measure and measure of maximal entropy along the central foliation of (conservative) Derived from Anosov (DA)

diffeomorphisms. We prove that for accessible DA diffeomorphisms of $\mathbb{T}^3$, atomic disintegration has the peculiarity of being mono-atomic (one atom per leaf). We further provide open and non-empty condition for the existence of atomic disintegration. Finally, we prove some new relations between Lyapunov exponents of DA diffeomorphisms and their linearization.

Group actions and shadowing

Sergey Tikhomirov

(Joint work with A. Osipov)

We introduce notion of shadowing property for actions of finitely generated not necessarily abelian groups. In contract with shadowing for diffeomorphisms and flows we show that shadowing property depends not only on hyperbolicity but on the group structure as well. For nilpotent groups we prove an analog of the shadowing lemma. We give an example of an action of a solvable group, whose shadowing property depends on quantitative properties of hyperbolicity. Finally we prove that there is no linear action of free nonabelian group which has shadowing property.

Lorenz attractors for flows and diffeomorphisms

Dmitri Turaev

We generalise the classical theory of Lorenz attractors based on two notions: volume-hyperbolicity and chain-transitivity. The theory extends to time-periodic perturbations of systems with a Lorenz attractor, to lattices of weakly coupled systems of this type, and other examples. We show that Newhouse wild hyperbolic sets and Bonatti-Diaz blenders can be a typical constituent of higher-dimensional analogues of the Lorenz attractor. We also discuss local and global bifurcations that create Lorenz attractors and their higher-dimensional analogues.

Geometric structure for entropy maximizing measures

Raul Ures

(Joint work with Marcelo Viana and Jiagang Yang)

We consider partially hyperbolic diffeomorphisms on 3-manifolds with one-dimensional compact center leaves. We show that, for most of such diffeomorphisms, the maximal measures can be associated to a skeleton, a set consisting of a finite number of saddles. As a

consequence we obtain that partially hyperbolic diffeomorphisms on 3D-nilmanifolds have at most two entropy maximizing measures that vary continuously.

Linear response formula and differentiability of thermodynamical properties

Paulo Varandas

(Joint work with A. Castro and T. Bomfim)

In this talk we discuss some recent contributions to the thermodynamical formalism of a robust class of non-uniformly expanding maps. We discuss the differentiability of the topological pressure and equilibrium states for potentials at high temperature in a nonuniformly expanding context. Some of the consequences include regularity of metric entropy, Lyapunov exponents and the large deviations rate function, for a large class of multidimensional non-uniformy expanding maps beyond the uniformly hyperbolic setting.

Good conditional measures for the center leaves of a Derived from Anosov (DA) Diffeomorphism

José Régis Varao

The idea is to prove that if f, a conservative DA partially hyperbolic on the 3-Torus, with C^1 center leaves and a uniform constant on the jacobians of the center holonomy, then f is C^1 conjugate to its linearization ([1]), in particular f is Anosov. The proof of the theorem goes through the construction of good conditional measures on the center leaves that contains good dynamical information. Quick motivation: It is known from [1] that even if the center foliation is C^1 (which implies absolute continuity) the diffeomorphism does not have to be C^1 conjugated to its linearization (in contrast to other known examples,). Hence, the goal is to see to what extent the behavior of the center foliation has to be strengned in order to obtain a rigidity result. And this is what is accomplished by the theorem presented above. If time allows some other related results from [1] will be presented to form a better picture.

[1] Regis Varao, Center foliation: absolute continuity, disintegration and rigidity. arXiv: 1302.1637, 2013.

A note on the Avila-Bochi-Herman formula

Carlos Vasquez

Let $(X, \mathcal{A}, \mu)$ be a probability space, $T : X \to X$ an ergodic transformation preserving μ and $A : X \to \mathbb{SL}(2, \mathbb{R})$ measurable such that $\log \|A\| \in L^1(\mu)$. M. Hermán, A. Ávila and J. Bochi considered the cocycle

$$(AR_\theta)(x) = A(x)R_\theta, \qquad x \in X,$$

where R_θ is a rotation of angle θ, they proved that

$$\frac{1}{2\pi}\int_0^{2\pi} \lambda^+(T, AR_\theta)\, d\theta = \int_X \log \frac{\|A(x)\| + \|A(x)\|^{-1}}{2}\, d\mu.$$

In this work we extend the previous result to a more general setting including matrices in $\mathbb{GL}(2, \mathbb{R})$. As application we consider a case of linear partially hyperbolic cocycle with zero central Lyapunov exponent. We exhibit a one parameter family of linear cocycle, isotopic to the original one such that for a positive measure set of parameters, the resulting cocycle is n on-uniformly hyperbolic. More general situations are also considered.

Non-wandering sets of interval skew products

Denis Volk

(Joint with Victor Kleptsyn)

We consider a class of skew products over transitive subshifts of finite type with interval fibers. For a natural class of 1-parameter families, we prove that for all but countably many parameter values the nonwandering set (in particular, the union of all attractors and repellers) has zero measure. As a consequence, the same holds for a residual subset of the space of skew products.

Equicontinuity and distality in foliation theory

Paweł Walczak

I shall report on recent ($\geqslant$ 2009) results on equicontinuous and distal pseudogroups, groups and foliations obtained by Jesus Alvarez Lopez, Andrzej Biś, Alberto Candel, Shigenori Matsumoto and myself.

Density and stability of ergodicity: diffeomorphisms with positive entropy

Amie Wilkinson

(Joint work with Artur Avila and Sylvain Crovisier)

We prove that C^1-generic volume-preserving diffeomorphisms with positive entropy (for the volume) are ergodic and non-uniformly hyperbolic.

Noncollision singularities in a simplified four-body problem

Jinxin Xue

(Joint work with Dmitry Dolgopyat)

In this work, we study a model of simplified four-body problem called planar two-center-two-body problem. In the plane, we have two fixed centers $Q_1 = (-\chi, 0), Q_2 = (0, 0)$ of masses 1, and two moving bodies Q_3 and Q_4 of masses $\mu \ll 1$. They interact via Newtonian potential. Q_3 is captured by Q_2, and Q_4 travels back and forth between two centers. Based on a model of Gerver, we prove that there is a Cantor set of initial conditions which lead to solutions of the Hamiltonian system whose velocities are accelerated to infinity within finite time avoiding all early collisions. We consider this model as a simplified model for the planar four-body problem case of the Painlevé conjecture. Time permits, I will talk about our recent work on the four-body problem.

On density theorems of three-dimensional vector fields

Dawei Yang

We would like to know the dynamics of the flow generated by typical vector fields. I will talk about the some of my recent results (with other collaborators) on this topic for three-dimensional vector fields, especially from domation to singular hyperbolicity.

Diffusive billiards

Hong Kun Zhang

I will first review the relation among diffusion processes, solutions of diffusion equations, and billiard systems, essentially though the Feynman-Kac formula. Major statistical properties for 2-d hyperbolic systems have been well understood recently. Several categories of

diffusive hyperbolic systems will be reviewed, which includes classical billiards, billiards under random or deterministic perturbations, systems with micro-geometric structure; time-dependent billiards. If the system has a physically interesting steady state, then it belong in the class of ergodic SRB measures and their modifications. Some Physical laws that are related to nonequilibrium billiards with physical origin will also be discussed.