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On some non-newtonian compressible flows

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Abstract

In this talk, we focus on compressible non-newtonian flows in the low weissenberg regime. We discuss the effect of different stress tensors.

On existence of weak solution to a model describing incompressible mixtures with thermal diffusion cross effects

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Abstract

We present a model describing unsteady flows of a heat conducting mixture composed from L constituents in two and three dimensional bounded domain. We assume that the flow of the mixture is described only by the barycentric velocity, and that the fluid is non-Newtonian. In addition, we assume that the diffusion flux depends also on the temperature gradient, describing the Soret effect, and that the heat flux depends also on the chemical potentials gradient, describing the Dufour effect. We briefly show under which assumptions on the constitutive equations the model obeys the first and the second laws of thermodynamics and for a large class of physically well-motivated constitutive relations we establish the existence of a weak solution. For simplicity we restrict ourselves only onto the linear models, i.e., the diffusion and the heat flux depend linearly on the temperature and chemical potentials gradients.

Robustness of regularity for a 3D Navier-Stokes system. A first step to numerical verification of regularity

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Abstract

We report our ongoing developments on a stability for an incompressible Navier-Stokes system in a 3D periodic box in fractional Sobolev spaces. It will serve as an analytical foundation of a numerical verification of regularity, inspired by [1, 2]. The analysis is done in spirit of a stability result of [3].

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Convergence of capillary fluid models: from the non-local to the local Korteweg system

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Abstract

In this talk we are interested in the compressible Navier-Stokes model endowed with a capillary term. After recalling fundamental results on global solutions for the compressible Navier-Stokes and local Korteweg systems in Besov spaces, we focus on models with non-local capillary term (depending on a small parameter). We prove global well-posedness for small data in the stable case and study the convergence, as the parameter tends to zero, towards the local Korteweg model in Besov spaces tailored to the non-local capillarity. For large data and without stability conditions we obtain local well-posedness.

The local well-posedness of Navier-Stokes-Poisson equations

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Abstract

We consider the Cauchy problem of the compressible Navier-Stokes equations with a Poisson-type potential in the whole space. This is a simplified hydrodynamical model describing the motion of a gaseous star. Using the Lagrangean method recently devised by R. Danchin and R. Danchin-P.B. Mucha to treat the system of density-dependent fluids, we prove the local existence and uniqueness of the solution, along with the continuity of data-solution map of the NSP equations.

Global weak solutions to the 1D compressible Euler equations with radiation

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Abstract

We consider the Cauchy problem for the equations of one-dimensional motions of a compressible inviscid gas coupled with radiation through a radiative transfer equation. Assuming suitable hypotheses on the transport coefficients and the data, we prove that the problem admits a weak solution. More precisely, we show that a sequence of approximate solutions constructed by a generalized Glimm's scheme admits a subsequence converging to an entropic solution of the problem.

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On the controllability of fluid flows with non-constant densities

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Abstract

The goal of this talk is to present some recent results on the local exact controllability of Navier-Stokes equations with non-homogeneous data for both incompressible and compressible cases. Especially, we will explain how to develop Carleman weights to deal simultaneously with the transport equation satisfied by the density together with the equation of the velocity. This talk is based on joint works with Mehdi Badra, Olivier Glass, Sergio Guerrero, Jean-Pierre Puel.

A controllability result for the the non-isentropic 1-D Euler equation

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Abstract

We examine the question of the boundary controllability of the one-dimensional non-isentropic Euler equation for compressible polytropic gas, in the context of weak entropy solutions. We consider the system in Eulerian coordinates and the one in Lagrangian coordinates. For both systems a result of controllability toward constant states is obtained (with a limitation on the adiabatic constant for the Lagrangian system). Moreover the solutions that are constructed remain of small total variation in space for all time.

Polymeric fluids

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Abstract

The essence of modelling polymeric fluids is encapsulated in the coupling of equations describing the evolution of macroscopic quantities (like velocity, pressure and eventually also density and temperature) with an additional equation describing the microscopic structure. The influence of the processes of polymerization and fragmentation will be accounted through the dependence of viscosity on the level of polymerization and/or appearance of the extra stress tensor. We concentrate on a dilute solution of polymer chains suspended in a non-Newtonian solvent, and we assumed that individual polymer chains do not interact with one another, but can be convected by the macroscopic velocity field, and are also subject to polymerization and fragmentation processes. The problem was to find a macroscopic velocity $v(t,x) : (0,T) \times \Omega \to \mathbb{R}^3$, a pressure $q(t,x) : (0,T) \times \Omega \to \mathbb{R}_+$ and a density of the polymer molecules of the length z namely $\mu(t,x,z) : (0,T) \times \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$ satisfying the following system of equations

$$\begin{aligned} v_t + v\nabla_x v + \nabla_x q - \mathrm{div}_x T &= f \\ \mathrm{div}_x v &= 0, \end{aligned} \tag{1}$$

where the stress tensor T is given by the formula

$$T(\mu, D_x v(t,x)) = \nu(\mu, |D_x v|) D_x v(t,x). \tag{2}$$

The quantity μ satisfies the following size-structure equation

$$\partial_t \mu + \mathrm{div}_x(v\mu) = \partial_z(\tau\mu) - B\mu + 2\int_z^\infty B(y)b(z,y)\mu(t,x,y)\, dy. \tag{3}$$

where $B(z) : \mathbb{R}_+ \to \mathbb{R}_+$ is the rate of fragmentation of polymer molecules, $b(z,y) : \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$ is the probability that a given particle of length y brakes into a particle of length z and $y - z$, and

$\tau(z) : \mathbb{R}_+ \to \mathbb{R}_+$ is the rate of coagulation of monomers. The system is complemented with boundary and initial conditions.

From the Korteweg system to the Euler Korteweg system

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Abstract

In this talk I will start with presenting some results on the local well-posedness of the Euler Korteweg system and I will make link with the Korteweg system via a evanescent vanishing viscosity limit process.

Derivation of compressible multiphase flow models

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Abstract

In this talk, we focus on compressible multiphase flows i.e, complex flows made of several compressible phases such that the melange occurs at a lower scale than observation. One classical way to model such flows is to write equilibrium equations for all phases separately and perform statistical means. This yields a coupled system of PDEs whose unknowns are the volumic fraction, density and velocity of each phase. One drawback of this derivation is that it requires a closure equation which is still controversial.
In this talk we focus on a rigorous derivation of multiphase flow models when all components share the same velocities. We shall explain how starting from compressible Navier Stokes system we might derive multi-component flows models by homogenization.

Dynamical stability of non-constant equilibria for the compressible Navier-Stokes equations in Eulerian coordinates

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Abstract

In this talk we show global existence and uniqueness of strong solutions to the isothermal compressible Navier-Stokes equations. The initial data have to be near equilibria which may be non-constant due to considering large external forces.
We are able to prove exponential stability of equilibria in the phase space and, above all, to study the problem in Eulerian coordinates. The crucial point for proving this result is a linearization that takes into account "structural properties" of the nonlinear problem.
Then the proof is based on a careful study of the associated linear problem.

On bounded solutions to the compressible isentropic Euler system

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Abstract

We consider the compressible isentropic Euler system in the whole space $\mathbb{R}^2$. Using the tools developed by De Lellis and Székelyhidi for incompressible Euler system we first prove that there exist Lipschitz initial data for which there exist infinitely many bounded admissible weak solutions. The proof is based on the analysis of the Riemann problem for the Euler system. Further study of the Riemann problem shows that for every Riemann initial data yielding the self-similar solution in the form of two admissible shocks there exist in fact infinitely many admissible bounded weak solutions. Moreover for some of these initial data such solutions dissipate more total energy than the self-similar solution which might be looked at as a natural candidate for the "physical" solution. Finally, we show that self-similar solutions consisting only of rarefaction waves are unique in the class of bounded admissible weak solutions.

Steady solutions to the barotropic Navier-Stokes system with singular pressure laws

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Abstract

We will sketch the proof of existence of weak solutions to the steady compressible Navier-Stokes system in the barotropic case for a class of pressure laws singular at vacuum. The problem is considered in a bounded domain in the plane with slip boundary conditions. The proof involves adapted version of ideas by Mucha and Pokorny. In particular, due to appropriate construction of approximate solutions, obtained density is shown to be bounded away from 0 (and infinity).

Some generalities about the numerical analysis of compressible Navier-Stokes and Stokes system

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Abstract

The aim of this talk is to present some generalities on the discretization of the compressible Navier-Stokes system. This scheme is based on Crouzeix-Raviart approximation spaces. The discretization of the momentum balance is obtained by the usual finite element technique. The discrete mass balance is obtained by a finite volume scheme. This scheme enjoys an unconditional stability property, in the sense that the energy and maximum-principle-based a priori estimates for the continuous problem also hold for the discrete solution. We will give some convergence result and a we will give an error estimate using the relative entropy.

Low Mach number limit and diffusion limit in a model of radiative flow

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Abstract

We consider an asymptotic regime for a simplified model of compressible Navier-Stokes-Fourier system coupled to the radiation, when hydrodynamical flow is driven to incompressibility through the low Mach number limit. We prove a global-in-time existence for the primitive problem in the framework of weak solutions and for the incompressible target system and we study the convergence of the primitive system toward its incompressible limit. Moreover, we investigate the cases when the radiative intensity is driven either to equilibrium or to non-equilibrium diffusion limit, depending the scaling performed, and we study the convergence of the system toward the aforementioned limits.

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Some topics in the mathematical thermodynamics of compressible fluids

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Abstract

We will talk about several issues related to the notions of weak solutions, dissipative solutions and stability properties to the compressible Navier-Stokes system and its approximations.

Cucker-Smale's flocking model with a singular communication weight

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Abstract

We are going to present the difficulties, problems and first results concerning the Cucker-Smale's flocking model with a singular communications weight. The Cucker-Smale's flocking model describes a collective self-driven motion of self-propelled particles that form a flock (such as flocks of birds, schools of fish). It is associated with a particle system governed by the following equations:

$$\dot{x}_i(t) = v_i(t),$$

$$\dot{v}_i(t) = \frac{1}{N}\sum_{j=1}^{N}\psi(|x_j - x_i|)(v_j - v_i),$$

where $x_i(t), v_i(t)$ are the position and velocity of the i-th particle at the time t and $\psi : [0,\infty) \to [0,\infty)$ is the said communication weight, that plays a role of the potential kernel.
We assume that $\psi(s) = s^{-\alpha}$ for some $\alpha \in (0,1)$ and since it is non-Lipschitz, many difficulties arise, that cannot be dealt with with general theory.

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Collective Motion and Bifluid Models

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Abstract

In this talk, I will present in one space dimension how to get models closed to those recently developped for collective motions. The talk will turn around singular compressible Navier-Stokes equations and on bifluid systems.

On a stationary compressible flow with slip-inflow boundary conditions

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Abstract

We investigate stationary flow of a compressible fluid in a cylindrical domain with inhomogeneous slip boundary conditions and nontrivial inflow/outflow. We show existence of strong solutions $(u, \rho) \in W_p^2 \times W_p^1$ in a vicinity of given laminar flow $(\bar{v}, \bar{\rho})$. The perturbed flow $(\bar{v}, \bar{\rho})$ can be a constant flow in the direction of the axis of the cylinder or a Poiseuille-like profile. It is well-known that, contrarily to the case of homogeneous boundary data, admission of inflow yields lack of compactness in the continuity equation, hence a direct application of a fixed point argument to the system linearized around $(\bar{v}, \bar{\rho})$ is not possible. In order to overcome this problem we introduce a kind of Lagrangian coordinates which straighten the characteristics of the continuity equation. Combined with appropried maximal regularity estimate, this method yields possible the application of the Banach fixed point theorem to show the existence of a solution. In the end I will discuss briefly other techniques which enable to overcome the problem of the lack of compactness. The talk is based on joint works with Piotr B. Mucha and Milan Pokorný.

A Linearized Model for Compressible Flow past a Rotating Obstacle: Analysis via Modified Bochner–Riesz Multipliers

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Abstract

We consider the flow of a compressible Newtonian fluid around or past a rotating rigid obstacle in R^3. After a coordinate transform to get a problem in a time-independent domain we assume the new system to be stationary. We linearize it and use Fourier transform to prove the existence of a unique solution in L^q-spaces. However, in contrast to the incompressible case with multipliers based on the heat kernel the new multiplier functions are related to Bochner-Riesz multipliers and require the restriction $6/5 < q < 6$.

On the global wellposedness of a free boundary problem for the Navier-Stokes equations in unbounded domains

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Abstract

In this talk, we would like to consider the global wellposedness of a free boundary problem for the Navier-Stokes equations in unbounded domains. The following two things are keys of our approach. First one is to derive decay properties of solutions for the homogeneous linearized system. Second one is to control nonlinear terms arising from boundary condition. We especially concentrate on the second one in this talk.

On some free boundary problem for the compressible barotropic viscous fluid flow

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Abstract

My talk is concerned with the free boundary value problem for a compressible barotropic viscous flow. I talk about a local in time existence theorem for any initial data in a uniformly $W_q^{2-1/q}$ domain Ω in the N dimensional Euclidean space ($N \geq 2$). The solution class is that $\rho \in W_p^1((0,T), L_q(\Omega)) \cap L_p((0,T), W_q^1(\Omega))$ and $u \in W_p^1((0,T), L_q(\Omega)) \cap L_p((0,T), W_q^2(\Omega))$ with $2 < p < \infty$ and $N < q < \infty$, where ρ and u are the mass density and velocity field, respectively.

And also, I would like to talk about a global in time existence theorem for initial data close to $(\rho_*, 0)$, where ρ_* is a positive constant describing the mass density of reference body. The key step in showing the local in time unique existence theorem is to prove the maximal L_p-L_q regularity theorem for the linearized equations. To obtain this theorem, I used the R-bounded solution operator to the generalized resolvent problem for the linearized equations together with Weis's operator valued Fourier multiplier theorem.

Multivalued equations motivated by granular flow model

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Abstract

We study the system describing flow of granular avalanches. The derivation of considered continuum flow models essentially bases on the fact that the characteristic length in the flowing direction is in general much larger than the thickness of an avalanche. Such an approach resulted in depth-averaged equation governed by generalized system of shallow water equations (Saint-Venant equations). The evolution of granular avalanches along an inclined slope is described by the mass and momentum conservation laws. Among the variety of models capturing the dynamics of granular flow, the Savage-Hutter model is one of the most commonly used frameworks. The model covers the process of fast moving avalanche, where the contribution of kinetic energy is significant. The system proposed by Savage and Hutter consists of conservation laws (conservation of mass and balance of momentum) describing the motion of a one-dimensional avalanche flow down a smoothly varying slope (hard surface)

$$\begin{aligned} \frac{\partial}{\partial t} h + \frac{\partial}{\partial x}(hv) &= 0 \\ \frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}\left(hv^2 + \frac{1}{2}\beta h^2\right) &= hg, \end{aligned} \tag{4}$$

where the dependent variables are the height $h : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$, and the velocity $v : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$. The height h is the thickness of the layer measured perpendicular to the given surface, and v is the mean velocity obtained by averaging over the thickness of the layer. The coefficient β defines the geometry of the hard surface. The right-hand side results from gravitation and friction, i.e. internal friction (viscosity) and friction between granular layer and the given surface.

We will discuss the approach of kinetic formulation to the above system.

The Oberbeck-Boussinesq approximation in R^3 as a limit of the compressible Naver-Stokes-Fourier system with a low Mach number

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Abstract

We will present the asymptotic analysis of solutions to the compressible Navier-Stokes-Fourier system, when the Mach number is small proportional to ϵ, Froud number is proportional to $\sqrt{\epsilon}$ and $\epsilon \to 0$ and the domain containing the fluid varies with changing parameter ϵ. In particular, the fluid is driven by a gravitation generated by object(s) placed in the fluid of diameter converging to zero. As $\epsilon \to 0$, we

will show that the fluid velocity converges to a solenoidal vector field satisfying the Oberbeck-Boussinesq approximation on R^3 space with a concentric gravitation force. The proof is based on the spectral analysis of the associated wave propagator (Neumann Laplacian) governing the motion of acoustic waves.

References

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Stability for compressible Navier-Stokes motions

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Abstract

We consider the motion of a compressible barotropic fluid in a bounded domain with nonslip boundary conditions governed by the compressible Navier-Stokes equations. Assume that there exist some special regular global solutions of the above problem. Our aim is to show existence of global regular solutions which at the initial time are close to the special solutions and remain close for all time. We use the energy method. This is the L_2 approach. As the special solutions we can choose spherically symmetric solutions. Then our domain is located between two spheres.