"Random Walk in Dynamic Random Environment"

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Random walk in random environment is a topic of major interest in mathematics, physics, chemistry and biology. Over the past 40 years it has been used to describe various types of diffusion processes in disordered media. It has been studied intensively at the microscopic, the mesoscopic and the macroscopic level, with the help of analytic, probabilistic and ergodic-theoretic techniques.

Both static and dynamic random environments have been of interest. In the static version the random environment once chosen is kept fixed, and the motion of the random walk depends on the environment it sees at its space (!) location. In the dynamic version the random environment evolves over time, and the motion of the random walk depends on the environment it sees at its space-time (!) location.

The key questions of interest are: What are the scaling properties of the random walk? How do these scaling properties depend on the law of the underlying random enviroment? What can be said about the constants appearing in the scaling laws? What is the role of dimension?

In this lecture series I present an overview of what has been achieved for random walk in dynamic random environment. The outline is as follows:

1. Background and motivation.
2. Main results achieved so far.
3. Main open problems.
4. Basic technique 1: Regerenation.
5. Basic technique 2: Renormalisation.

Most results in the literature require mixing properties of the random environment in space, respectively, space-time. Part of the lecture series is geared towards understanding the precise role of these mixing properties.