Hadamard type operators for real analytic functions of several variables and moments of analytic functionals

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Abstract:
Any linear (continuous) map $M : A(\Omega) \to A(\Omega)$ such that all monomials $x^\alpha := x_1^{\alpha_1} \dots x_d^{\alpha_d}$ are eigenvectors is called a (Hadamard type) multiplier on the space $A(\Omega)$ of real analytic functions over an open set $\Omega \subset \mathbb{R}^d$. By (m_α) we denote the corresponding multisequence of eigenvalues which uniquely identifies the operator M.

We show that multipliers M correspond via an explicit formula to analytic functionals T (i.e., linear continuous functionals on a suitable space of real analytic functions) so that the sequence of eigenvalues (m_α) corresponds to the sequence of moments of the analytic functional $(\langle T, x^\alpha \rangle)_\alpha$. We provide several examples of multipliers and corresponding analytic functionals.

Then we describe sequences (m_α) of moments of analytic functionals in terms of holomorphic functions interpolating values m_α at points in the positive integer lattice $\mathbb{N}^d$ — the latter result belongs to the classical line of ideas on interpolation of Taylor coefficients of functions of one variable by holomorphic functions of restricted growth. This characterization allows us to get sufficient and necessary conditions on multipliers to be surjective. In consequence, we get several examples of linear partial differential operators of polynomial coefficients (Euler type operators) which are surjective or which are not surjective on the space $A(\mathbb{R}^d)$.

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