

ALMOST OVERCOMPLETE AND ALMOST OVERTOTAL SEQUENCES IN BANACH SPACES

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Abstract

The new concepts are introduced of almost overcomplete sequence in a Banach space and almost overttotal sequence in a dual space.

Definition 0.1. *A sequence in a Banach space X is said to be almost overcomplete whenever the closed linear span of any its subsequence has finite codimension in X .*

Definition 0.2. *Let X be a Banach space. A sequence in the dual space X^* is said to be almost overttotal on X whenever the annihilator (in X) of any its subsequence has finite dimension.*

The following theorem is our main result.

Theorem 0.3. *Each almost overcomplete bounded sequence in a Banach space, and each almost overttotal bounded sequence in a dual Banach space is relatively norm-compact.*

We give several applications of this theorem. For instance

Theorem 0.4. *Let X, Y be infinite-dimensional Banach spaces, Y having an unconditional basis $\{u_i\}_{i=1}^{\infty}$ with $\{e_i\}_{i=1}^{\infty}$ as the sequence of the associated coordinate functionals. Let $T : X \rightarrow Y$ be a one-to-one bounded non compact linear operator. Then there exist an infinite-dimensional subspace $Z \subset X$ and a strictly increasing sequence $\{k_l\}$ of integers such that $e_{k_l}(Tz) = 0$ for any $z \in Z$ and any $l \in \mathbb{N}$.*