Mean-width and mean-norm of isotropic convex bodies

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We discuss upper bounds for the mean-width $M^*(K) = \int_{S^{n-1}} h_K(x)\, d\sigma(x)$ and the mean-norm $M(K) = \int_{S^{n-1}} \|x\|\, d\sigma(x)$ of an isotropic symmetric convex body K in $\mathbb{R}^n$; here, h_K is the support function of K and $\|\cdot\|$ is its induced norm on $\mathbb{R}^n$. An essentially optimal upper bound for $M^*(K)$ was obtained recently by Emanuel Milman; he showed that

$$M^*(K) \leqslant C\sqrt{n}\log^2 n\, L_K,$$

where L_K is the isotropic constant of K. In a joint work with E. Milman, we have shown that

$$M(K) \leqslant \frac{C\log^{2/5}(e+n)}{\sqrt[10]{n}L_K}.$$

Both results follow from analogous estimates for the mean-width and the mean-norm of the L_q-centroid bodies of K. These, in turn, are based on a more general scheme; for example, we show that if K is a centrally-symmetric convex body in $\mathbb{R}^n$ with $K \supseteq rB_2^n$ then

$$\sqrt{n}M(K) \leqslant C\sum_{k=1}^{n} \frac{1}{\sqrt{k}} \min\left(\frac{1}{r}, \frac{n}{k}\log\left(e+\frac{n}{k}\right)\frac{1}{v_k^-(K)}\right).$$

where $v_k^-(K)$ denotes the minimal volume-radius of a k-dimensional orthogonal projection of K.

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