

# Hyperplane inequalities for measures of convex bodies

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The hyperplane problem asks whether there exists an absolute constant  $C$  so that for any origin-symmetric convex body  $K$  in  $R^n$

$$|K|^{\frac{n-1}{n}} \leq C \max_{\xi \in S^{n-1}} |K \cap \xi^\perp|,$$

where  $\xi^\perp$  is the central hyperplane in  $R^n$  perpendicular to  $\xi$ , and  $|K|$  stands for volume of proper dimension. The problem is still open, with the best-to-date estimate  $C \sim n^{1/4}$  established by Klartag, who slightly improved the previous estimate of Bourgain. It is much easier to get a weaker estimate with  $C = \sqrt{n}$ .

In this talk we show that the  $\sqrt{n}$  estimate holds for arbitrary measure in place of volume. Namely, if  $L$  is an origin-symmetric convex body in  $R^n$  and  $\mu$  is a measure with non-negative even continuous density on  $L$ , then

$$\mu(L) \leq \sqrt{n} \frac{n}{n-1} c_n \max_{\xi \in S^{n-1}} \mu(L \cap \xi^\perp) |L|^{1/n},$$

where  $c_n = |B_2^n|^{\frac{n-1}{n}} / |B_2^{n-1}| < 1$ , and  $B_2^n$  is the unit Euclidean ball in  $R^n$ . We deduce this inequality from a stability result for intersection bodies. We also present lower dimensional and complex versions of this inequality. We also prove that  $\sqrt{n}$  can be replaced by an absolute constant for some special classes of convex bodies, in particular, for unconditional convex bodies,  $k$ -intersection bodies,  $1 \leq k < n$  (the unit ball of a finite dimensional space that embeds in  $L_{-k}$ ; the constant depends only on  $k$ ), and duals of bodies with bounded volume ratio.