

Pointwise products, pointwise multipliers and factorization

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Given a couple of Banach function spaces X and Y over the same measure space, the space of *pointwise multipliers* is defined as

$$M(X, Y) = \{y \in L^0 : yx \in Y \quad \forall x \in X\}$$

with the usual operator norm and *pointwise product* is just

$$X \odot Y = \{xy : x \in X, y \in Y\}$$

with the quasi norm $\|z\|_{X \odot Y} = \inf\{\|x\|_X \|y\|_X : z = xy\}$. Using this notion, Lozanovskii's factorization theorem may be written as $X \odot M(X, L^1) \equiv L^1$. We shall discuss the following problem: when there holds

$$X \odot M(X, Y) = Y?$$

This is, when Y may be factorized through X ? We will discuss some known positive and negative results of such a type, as well as present a new results on factorization of concrete classes of Banach function spaces including Calderón - Lozanovskii, Orlicz, Lorentz and Marcinkiewicz spaces. Finally, some general relations between constructions $M(X, Y)$ and $X \odot Y$ will be discussed. The talk is based on two papers:

- [KLM13] P. Kolwicz, K. Leśnik and L. Maligranda, *Pointwise multipliers of Calderón-Lozanovskii spaces*, Math. Nachr. 286 (2013), 876–907.
- [KLM14] P. Kolwicz, K. Leśnik and L. Maligranda, *Pointwise products of some Banach function spaces and factorization*, J. Funct. Anal. 266 (2014), no. 2, 616–659.