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Structure of Cesàro function spaces and interpolation

Abstract

The *Cesàro function spaces* $Ces_p(I)$ on both $I = [0, 1]$ and $I = [0, \infty)$ are classes of Lebesgue measurable real functions f on I such that the norm $\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x}\int_0^x |f(t)|dt\right)^p \ dx\right]^{1/p} < \infty$ for $1 \leq p < \infty$ and $\|f\|_{C(\infty)} = \sup_{x \in I,\ x>0} \frac{1}{x}\int_0^x |f(t)|\,dt < \infty$ for $p = \infty$. In the case $1 < p < \infty$ spaces $Ces_p(I)$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property.

The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0, 1]$ and $[0, \infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \leq q \leq \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0, 1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0, 1]$ and $[0, \infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0, 1]$ for $1 \leq p < \infty$ a space which is isomorphic to l^2. This subspace is uncomplemented in $Ces_p[0, 1]$. The span in the space $Ces_\infty[0, 1]$ gives another sequence space.

In [4] and [5] it was shown that $Ces_p(I)$ is an interpolation space between $Ces_{p_0}(I)$ and $Ces_{p_1}(I)$ for $1 < p_0 < p_1 \leq \infty$, where $1/p = (1 - \theta)/p_0 + \theta/p_1$ with $0 < \theta < 1$. The same result is true for Cesàro sequence spaces. On the other hand, $Ces_p[0, 1]$ is not an interpolation space between $Ces_1[0, 1]$ and $Ces_\infty[0, 1]$.

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