Traces and spectral properties of shift operators.

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Abstract:
The Calkin theorem says that there is a one-to-one correspondence between all ideals of operators on the separable infinite-dimensional Hilbert space H and all symmetric ideals of bounded sequences (σ_n). Passing (in a suitable way) to dyadic subsequences (σ_{2^k}) with $k \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$, we obtain a new class of sequence ideals $\mathfrak{z}(\mathbb{N}_0)$, which are shift-invariant and monotone. The latter property means that every $(\alpha_k) \in \mathfrak{z}(\mathbb{N}_0)$ is dominated by a monotone $(\beta_k) \in \mathfrak{z}(\mathbb{N}_0)$. That is, $|\alpha_k| \leq \beta_k$ and $\beta_0 \geq \beta_1 \geq \beta_2 \geq \dots \geq 0$. This procedure reduces the theory of traces on operator ideals $\mathfrak{A}(H)$ to the theory of shift-invariant linear forms on the associated sequence ideal $\mathfrak{z}(\mathbb{N}_0)$. Criterions for the existence of various kinds of traces on $\mathfrak{A}(H)$ can be stated in terms of spectral properties of the shift operators on $\mathfrak{z}(\mathbb{N}_0)$. We present a classification of operator ideals from the trace-theoretical standpoint.