

# Quantum expanders and $C^*$ -norms on tensor products of von Neumann algebras

by

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## Abstract

This is mainly an account of joint work with N. Ozawa. For any pair  $M, N$  of von Neumann algebras such that the algebraic tensor product  $M \otimes N$  admits more than one  $C^*$ -norm, the cardinal of the set of  $C^*$ -norms is at least  $2^{\aleph_0}$ . Moreover there is a family with cardinality  $2^{\aleph_0}$  of injective tensor product functors for  $C^*$ -algebras in Kirchberg's sense. Let  $\mathbb{B} = \prod_n M_n$ . We also show that, for any non-nuclear von Neumann algebra  $M \subset \mathbb{B}(\ell_2)$ , the set of  $C^*$ -norms on  $\mathbb{B} \otimes M$  has cardinality *equal to*  $2^{2^{\aleph_0}}$ . The talk will also recall the connection of such questions with the non-separability of the set of finite dimensional (actually even of 3-dimensional) operator spaces which goes back to a 1995 paper with Marius Junge, and several recent "quantitative" refinements obtained using an estimate of the metric entropy of the set of quantum expanders.