Smooth function spaces on self-similar sets

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Abstract

It is shown that complemented subspaces of s, that is, nuclear Fréchet spaces with properties (DN) and Ω, which are 'almost normwise isomorphic' to a multiple direct sum of copies of themselves are isomorphic to s. Basic ingredients of the proof are calculation of the diametral dimension and the Pełczyński decomposition method. The result is applied, for instance, to spaces of smooth functions on the Cantor set or the Sierpiński triangle. For these sets it is shown that $C_\infty(X) \cong s$ where $C_\infty(X)$ is the space of restrictions of C^∞-functions to X. If X is the classical Cantor set it is, moreover, shown that $A_\infty(X) = C_\infty(X) \cong s$, so solving another open problem. Here $A^\infty(X)$ denotes the space restrictions of 2π-periodic C^∞-functions on $\mathbb{R}$ with vanishing negative Fourier coefficients or, equivalently, the space of holomorphic functions on the unit disc with C^∞-boundary values.